Queen Mary
University of London

B. Sc. Examination by Course Unit 2005

MAS 108 Probability I

Duration: 2 hours

Date and time: 13 May 2005, 10:00–12:00

The paper has two Sections and you should attempt both Sections. Please read carefully the instructions given at the beginning of each Section.

Calculators are NOT permitted in this examination. The unauthorised use of a calculator constitutes an examination offence.

The New Cambridge Elementary Statistical Tables are provided.

Section A: You should attempt all questions. Marks awarded are shown next to the questions. This part of the examination carries 60% of the marks.

Question 1 (5 marks) A box contains five tickets labelled 1, 2, 3, 4, 5. Three tickets are drawn at random without replacement. Write down the sample space, assuming that the order in which the tickets are drawn does not matter. [5]

Question 2 (5 marks) The probability that a random train at Stepney Green station is a District Line train is $3/4$. (The remaining trains are Hammersmith & City Line). The probability that the train is running late is $1/6$. The probability that it is a District Line train running late is $1/12$. Find the probability that a random train is

(a) a District Line train which is not running late; [2]

(b) a Hammersmith & City Line train which is not running late. [3]

Question 3 (12 marks)

(a) State Kolmogorov's Axioms for probability. [4]

(b) Prove from Kolmogorov's Axioms the *Inclusion–Exclusion Rule* for two events A and B, namely $P(A \cup B) = P(A) + P(B) - P(A \cap B)$. [6]

(c) State without proof the Inclusion–Exclusion Rule for three events A, B and C. [2]

Question 4 (10 marks)

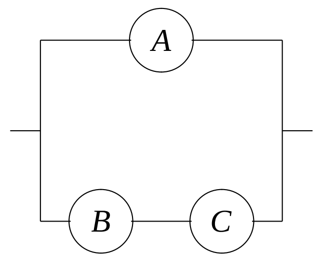
(a) X is a Poisson random variable with parameter 2.4.

(i) Find $P(X = 0)$. [2]

(ii) Find $P(1 \leq X \leq 3)$. [2]

(b) Y is a Binomial random variable with parameters $(48, \frac{1}{4})$. By making a suitable approximation (which you should state clearly), find $P(11 \leq Y \leq 16)$. [6]

Question 5 (6 marks) The electrical apparatus in the diagram works so long as current can flow from left to right. The three components are independent. The probability that component A works is 0.75; the probability that component B works is 0.7; and the probability that component C works is 0.8.



Find the probability that the apparatus works. [6]

Question 6 (8 marks)

(a) Define the *expected value* and the *variance* of a discrete random variable. [2]

(b) Let the probability mass function of the random variable X be given by the following table:

x	1	2	3	4
$P(X = x)$	0.4	0.3	0.2	p

Find p. Hence find the expected value and the variance of X. [6]

Question 7 (10 marks) The continuous random variable X has cumulative distribution function $F_X(x)$ given by

$$F_X(x) = \begin{cases} 0 & \text{if } x < 0; \\ x^3 & \text{if } 0 \leq x \leq 1; \\ 1 & \text{if } x > 1. \end{cases}$$

(a) Find $P(0 < X < 0.5)$. [2]

(b) Find the probability density function of X. [3]

(c) Find the expected value of X. [3]

(d) Find the median of X. [2]

Question 8 (4 marks) Give an example of a practical situation which could be modelled by a *geometric* random variable. You should explain clearly what quantity the random variable models, and what the parameters of its distribution are. [4]

Section B: You may attempt as many questions as you wish and all questions carry equal marks. Except for the award of a bare pass, only the best TWO questions answered will be counted. This part of the examination carries 40% of the marks.

Question 9 (20 marks) I have a collection of textbooks to sell, and advertise for offers. I decide to accept any offer over 100 pounds.

However, if the first offer is less than 100 pounds, then I will subsequently accept any offer over 80 pounds.

The offers I receive are normally distributed with expected value 80 and variance 64, and are mutually independent.

(a) Find the probability that I accept the first offer for my textbooks. [5]

(b) Find the probability that I accept one of the first two offers. [5]

(c) Find the expected number of offers I receive, up to and including the one I accept. [10]

(Give your answers to four places of decimals.)

Question 10 (20 marks)

Define the *covariance*, and the *correlation coefficient*, of two discrete random variables X and Y. [3]

Prove that, if X and Y are independent, then $\mathrm{Cov}(X,Y) = 0$. Give an example to show that the converse is false. [9]

Let X and Y be Bernoulli random variables with parameters p_1 and p_2 respectively, and let $P(X = 1, Y = 1) = r$. Prove that $\mathrm{Cov}(X,Y) = r - p_1 p_2$. Hence show that, if $\mathrm{Cov}(X,Y) = 0$, then X and Y are independent. [5]

Now suppose that $p_1 = p_2 = p$, say. Show that the correlation coefficient of X and Y is equal to 1 if and only if $P(X = Y) = 1$. [3]

Question 11 (20 marks) Let X be an exponential random variable with parameter λ. Write down the probability density function of X, and use this to find the cumulative distribution function of X. [6]

Now suppose that X_1 and X_2 are independent exponential random variables with parameters λ_1 and λ_2 respectively, and let $Y = \min\{X_1, X_2\}$. For any real number x, show that $Y \geq x$ if and only if $X_1 \geq x$ and $X_2 \geq x$. Hence show that Y is also an exponential random variable, and find its parameter. [12]

Let $Z = \max\{X_1, X_2\}$. FInd the c.d.f. of Z. [2]

Question 12 (20 marks) For a pregnant woman aged 30, the probability that her child will be born with Down's syndrome is $1/300$.

(a) A blood test can be performed to see if the child will be born with Down's syndrome. If the foetus has Down's syndrome then the probability that the blood test is positive is $7/10$; otherwise the probability that the blood test is positive is $2/10$.

A pregnant woman aged 30 has the blood test.

(i) Find the probability that the result is negative. [5]

(ii) Find the conditional probability that the child will have Down's syndrome given that the result of the blood test is negative. [3]

(b) Another type of test is an ultrasound scan. If the foetus has Down's syndrome then the probability that the scan is positive is $8/10$; otherwise the probability that the scan is positive is $1/10$.

Find the conditional probability that the child will have Down's syndrome given that the result of the scan is negative. [6]

(c) Now suppose that the two tests are independent. This means that, no matter whether the child has Down's syndrome or not, the conditional probability that both tests are negative is equal to the conditional probability that the blood test is negative multiplied by the conditional probability that the scan is negative.

Find the conditional probability that the child will have Down's syndrome given that the results of both tests are negative. [6]

 END OF EXAMINATION