Queen Mary
University of London

MAS 108 — Probability I

Discrete random variables — Summary

Bernoulli random variable Bernoulli(p)

- Occurs when there is a single trial with a fixed probability p of success.
- Takes only the values 0 and 1.
- p.m.f. $P(X=0)=q$, $P(X=1)=p$, where $q=1-p$.
- $E(X)=p$, $\mathrm{Var}(X)=pq$.
- The indicator function of an event is a Bernuolli random variable.

Binomial random variable $\mathrm{Bin}(n,p)$ (Lindley and Scott, Table 1)

- Occurs when we are counting the number of successes in n independent trials with fixed probability p of success in each trial, e.g. the number of heads in n coin tosses. Also, sampling with replacement from a population with a proportion p of distinguished elements.
- The sum of n independent Bernoulli(p) random variables.
- Values 0, 1, 2, $\ldots$, n.
- p.m.f. $P(X=m)={}^{n}\mathrm{C}_{m}q^{n-m}p^{m}$ for $0\leq m\leq n$, where $q=1-p$.
- $E(X)=np$, $\mathrm{Var}(X)=npq$.

Geometric random variable Geom(p)

- Describes the number of trials up to and including the first success in a sequence of independent Bernoulli trials, e.g. number of tosses until the first head when tossing a coin.
- Values 1, 2, ... (any positive integer).
- p.m.f. $P(X = m) = q^{m-1}p$, where $q = 1 - p$.
- $E(X) = 1/p$, $\text{Var}(X) = q/p^2$.

Hypergeometric random variable Hg(n, M, N)

- Occurs when we are sampling n elements without replacement from a population of N elements of which M are distinguished. The random variable is the number of distinguished elements in the sample.
- Values 0, 1, 2, ..., n (except that the smallest value is $n - N + M$ if $N - M < n$ and the largest value is M if $M < n$).
- p.m.f. $P(X = m) = ({}^M\mathrm{C}_m \cdot {}^{N-M}\mathrm{C}_{n-m})/{}^N\mathrm{C}_n$.
- $E(X) = n\left(\frac{M}{N}\right)$, $\text{Var}(X) = n\left(\frac{M}{N}\right)\left(\frac{N-M}{N}\right)\left(\frac{N-n}{N-1}\right)$.
- Approximately Bin($n, M/N$) if n is small compared to $N, M, N - M$.

Poisson random variable Poisson(λ) (Lindley and Scott, Table 2)

- Describes the number of occurrences of a random event in a fixed time interval, e.g. the number of fish caught in a day.
- Values 0, 1, 2, ... (any non-negative integer)
- p.m.f. $P(X = m) = \mathrm{e}^{-\lambda}\lambda^m/m!$.
- $E(X) = \lambda$, $\text{Var}(X) = \lambda$.
- If n is large, p is small, and $np = \lambda$, then Bin(n, p) is approximately Poisson(λ).

D. V. Lindley and W. F. Scott, *New Cambridge Statistical Tables*, Cambridge University Press.