Queen Mary
University of London

# MAS 108 — Probability I

## Continuous random variables — Summary

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### Uniform random variable $U[a,b]$

- Occurs when a number is chosen at random from the interval $[a,b]$, with all values equally likely.
- p.d.f. $f(x) = \begin{cases} 0 & \text{if } x < a, \\ 1/(b-a) & \text{if } a \le x \le b, \\ 0 & \text{if } x > b. \end{cases}$
- c.d.f. $F(x) = \begin{cases} 0 & \text{if } x < a, \\ (x-a)/(b-a) & \text{if } a \le x \le b, \\ 1 & \text{if } x > b. \end{cases}$
- $E(X) = (a+b)/2$, $\mathrm{Var}(X) = (b-a)^2/12$.

### Normal random variable $N(\mu,\sigma^2)$ (Lindley and Scott, Table 4)

- The limit of the sum (or average) of many independent Bernoulli random variables. This also works for many other types of random variables: this statement is known as the *Central Limit Theorem*.
- p.d.f. $f(x) = \dfrac{1}{\sigma\sqrt{2\pi}}\,\mathrm{e}^{-(x-\mu)^2/2\sigma^2}$.
- No simple formula for c.d.f.; use tables.
- $E(X) = \mu$, $\mathrm{Var}(X) = \sigma^2$.
- For large $n$, $\mathrm{Bin}(n,p)$ is approximately $N(np,npq)$.
- *Standard normal* $N(0,1)$ is given in the table. For other normal random variables, use the fact that if $X \sim N(\mu,\sigma^2)$, then $(X-\mu)/\sigma \sim N(0,1)$.

**Exponential random variable** $\text{Exp}(\lambda)$

- Occurs in the same situations as the Poisson random variable (see summary sheet on Discrete Random Variables), but measures the time from now until the first occurrence of the event.
- p.d.f. $f(x) = \begin{cases} 0 & \text{if } x < 0, \\ \lambda\ \mathrm{e}^{-\lambda x} & \text{if } x \geq 0. \end{cases}$
- c.d.f. $F(x) = \begin{cases} 0 & \text{if } x < 0, \\ 1 - \mathrm{e}^{-\lambda x} & \text{if } x \geq 0. \end{cases}$
- $E(X) = 1/\lambda$, $\text{Var}(X) = 1/\lambda^2$.
- However long you wait without anything happening, the time until the next occurrence has the same distribution.

D. V. Lindley and W. F. Scott, *New Cambridge Statistical Tables*, Cambridge University Press.