Queen Mary
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MAS 108 Probability I

Solution to challenge problem Autumn 2005

(a) Let X_1 be the number of cereal packets needed to get a photo of the first lecturer, and let X_2 be the number of cereal packets needed after that to get a photo of the second lecturer (here 'first' means the first one whose photo you get, not necessarily the first one I named).

Then $N = X_1 + X_2$ so $E(N) = E(X_1) + E(X_2)$.

But $X_1 = 1$ so $E(X_1) = 1$ so $E(N) = 1 + E(X_2)$.

After the first photo has been obtained, each cereal packet has probability $1/2$ of containing the other, so $X_2 \sim \text{Geom}(1/2)$ and so $E(X_2) = 2$.

Therefore $E(N) = 1 + 2 = 3$.

(b) As above, $E(M) = 1 + E(X_2)$.

To find $E(X_2)$, we need to condition on the first photo. Let S be the event that the first photo is of Dr. Soicher. Then $P(S) = 9/10$ and $P(S') = 1/10$.

Now, $X_2 \mid S \sim \text{Geom}(1/10)$ so $E(X_2 \mid S) = 10$, and $X_2 \mid S' \sim \text{Geom}(9/10)$ so $E(X_2 \mid S') = 10/9$. Therefore

$$\begin{aligned} E(X_2) &= P(S)E(X_2 \mid S) + P(S')E(X_2 \mid S') \\ &= \frac{9}{10} \times 10 + \frac{1}{10} \times \frac{10}{9} = \frac{82}{9}. \end{aligned}$$

Hence

$$E(M) = 1 + \frac{82}{9} = \frac{91}{9}.$$

(c) If you want collectors to buy a large number of packets, make one of the items have a much lower probability than the rest.