

QUEEN MARY, UNIVERSITY OF LONDON

MAS 108

Probability I

Assignment 8

For handing in on 28 November 2005

Write your name and student number at the top of your assignment before handing it in. Staple all the pages together. Post the assignment in the red post-box on the ground floor of the Maths building before 1600 on Monday.

This week's reading: Devore, Chapter 3, Sections 3.5–3.6, and Chapter 4, Sections 4.1–4.2, and Table A.2; or Hines and Montgomery, Chapter 3, Sections 3.2, 3.4 and 3.5, and Chapter 6, Sections 6.7–6.9; or Rice, Chapter 2, Sections 2.1–2.2, and Chapter 4, Sections 4.1–4.2. Also New Cambridge Statistical Tables, Table 2.

1 (10 marks) There are 24 unshorn sheep in a field. The farmer shears six of the sheep and returns them to the field. The next day he randomly catches five sheep from that field. He counts how many of these sheep are shorn. Assume that no sheep leave or enter the field, or die or give birth, between the shearing and the capture. Let X be the number of shorn sheep among those caught on the second day.

Without using the probability mass function for X , calculate $E(X)$ and $\text{Var}(X)$ as exact fractions.

2 (20 marks) The distribution of the number of text messages received on Emma's mobile phone in a 10-minute interval is approximately Poisson with mean $1/4$. Find

- (a) the probability that a text message arrives during that 10-minute interval;
- (b) the probability that exactly one text message is received on Emma's mobile in that 10-minute interval.

3 (20 marks) (a) Use the Poisson approximation to give an approximate value for $P(X \leq 3)$ when $X \sim \text{Bin}(200, 0.015)$.

- (b) There are 10,000 buttons in a large box, 600 of which are red. I put my hand into the box and randomly take out 20 buttons. By making an appropriate approximation, find the probability that exactly 3 of the buttons in my hand are red.

4 (30 marks) Let X be a continuous random variable whose probability density function f is given by

$$f(x) = \begin{cases} 0 & \text{if } x < 4 \\ \theta & \text{if } 4 \leq x \leq 10 \\ 0 & \text{if } x > 10 \end{cases}$$

for some constant θ .

- (a) Find the value of θ .
- (b) Find the cumulative distribution function F of X .
- (c) Find
 - (i) $P(X \leq 8)$,
 - (ii) $P(X \geq 5)$,
 - (iii) $P(X \geq 12)$
 - (iv) $P(7 \leq X \leq 9)$.
- (d) Find $E(X)$.
- (e) Find $\text{Var}(X)$.

5 (20 marks) Let X be a continuous random variable whose cumulative distribution function F is given by

$$F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1 - \cos x}{2} & \text{if } 0 \leq x \leq \pi \\ 1 & \text{if } \pi < x \end{cases}$$

- (a) Find the probability density function of X .
- (b) Find $E(X)$.
- (c) Find the median of X .
- (d) Find the upper quartile of X .