QUEEN MARY, UNIVERSITY OF LONDON

MAS 108 Probability I

Assignment 5   For handing in on 31 October 2005

Write your name and student number at the top of your assignment before handing it in. Staple all the pages together. Post the assignment in the red post-box on the ground floor of the Maths building before 1600 on Monday.

This week's reading: Devore, Chapter 2, Section 2.4; *or* Hines and Montgomery, Chapter 2, Section 2.8; *or* Rice, Chapter 1, Section 1.5.

1 (Animal Health, continued again) (20 marks) Suppose that the original bottle of liquid contains exactly three germs (infectious units). Let

$$
\begin{aligned}
A_1 &= \text{``exactly one aliquot is infectious''} \\
A_2 &= \text{``exactly two aliquots are infectious''} \\
A_3 &= \text{``exactly three aliquot are infectious''} \\
B &= \text{``none of the animals becomes ill''}.
\end{aligned}
$$

In Assigment 3, Question 5, we calculated that $P(B \mid A_2) = 3/7$. In Assigment 4, Question 2, we calculated that $P(A_1) = 1/225$, $P(A_2) = 42/225$ and $P(A_3) = 182/225$. Find $P(B)$.

2 (Continued from Assignment 4) (30 marks) We know that if a person in the general population does not have cystic fibrosis then the probability that they have one C gene is $2/51$. Suppose that the QMUL Medical School has developed a test for the C gene: 95% of people who have one C gene give a positive test result, while only 2% of those with no C gene give a positive test result. An individual is chosen at random from among people without cystic fibrosis, and is tested. Find the conditional probability that

(a) the person has one C gene given that the test is positive;

(b) the person has no C gene given that the test is negative.

Give your answers to 4 decimal places.

TURN OVER FOR NEXT QUESTION

3 (50 marks) John has just passed four of his eight first-year exams and failed the other four, so he guesses that he has probability $1/2$ of passing any exam, independently of all other exams and independently of how often he has failed it in the past. Let us assume that his guess is correct.

  (a) To qualify for the second year, he has to pass at least six of his eight first-year courses. At his first resit opportunity he takes all four of the exams which he failed. Find the probability that he qualifies for the second year after his first resits.

  (b) If he still does not qualify for the second year, he is allowed a second (and last) resit opportunity. Find the conditional probability that he qualifies for the second year after his second resits given that he did not qualify after the first resits.

  (c) Overall, what is the probability that he qualifies for the second year?

  (d) Suppose that you know that he passed four exams out of eight at the first attempt and that he eventually qualified for the second year. What is the probability that he failed everything in the first resits?