QUEEN MARY, UNIVERSITY OF LONDON

MAS 108 Probability I

Assignment 4 For handing in on 24 October 2005

Write your name and student number at the top of your assignment before handing it in. Staple all the pages together. Post the assignment in the red post-box on the ground floor of the Maths building before 1600 on Monday.

This week's reading: Devore, Chapter 2, Section 2.4; *or* Hines and Montgomery, Chapter 2, Section 2.7; *or* Rice, Chapter 1, Section 1.5.

1 (10 marks) Consider Question 6 of Assignment 2.

(a) Find $P(F \mid M)$.

(b) Find the conditional probability that there is a defect in the frame given that there is a defect anywhere in the bicycle.

2 (Animal Health, continued) (20 marks) A bottle of liquid contains an unknown number of infectious units. The scientists want to know how many. This is how they try to find out. First, they mix the liquid thoroughly and divide it into 15 equal portions, called *aliquots*. An aliquot is said to be *infectious* if it contains any of the infectious units. Then five aliquots are chosen at random. Each chosen aliquot is injected into one animal. If the aliquot is infectious then the animal will become ill; otherwise the animal remains healthy. The scientists count how many of the five animals become ill.

Suppose that initially there are exactly three infectious units in the liquid. When the liquid is mixed and divided into aliquots, each infectious unit is equally likely to end up in any of the fifteen aliquots, independently of all the other infectious units.

(a) Find the probability that only one of the aliquots becomes infectious (i.e. that it contains all of the infectious units).

(b) Find the probability that three of the aliquots become infectious (i.e. that the three infectious units end up in different aliquots).

(c) Find the probability that exactly two of the aliquots become infectious.

3 (15 marks) Two electrical components G and H are connected in parallel. Independently of each other, $P(G \text{ works}) = 3/4$ and $P(H \text{ works}) = 3/5$. Current flows if either component works.

Find $P(G \text{ is not working} \mid \text{current is flowing})$.

4 (15 marks) I have three fair coins in my pocket: one has heads on one side and tails on the other; the second has heads on both sides; the third has tails on both sides.

I take a coin out of my pocket at random and toss it. It comes down heads. What is the probability that its other side is heads?

5 (40 marks) Each person has two genes for cystic fibrosis. Each gene is either N or C. Each child receives one gene from each parent. If your genes are NN or NC or CN then you are normal; if they are CC then you have cystic fibrosis.

(a) Neither of Sally's parents has cystic fibrosis. Nor does she. However, Sally's sister Hannah does have cystic fibrosis. Find the probability that Sally has at least one C gene (given that she does not have cystic fibrosis).

(b) In the general population the ratio of N genes to C genes is about 49 to 1. You can assume that the two genes in a person are independent. Michael does not have cystic fibrosis. Find the probability that he has at least one C gene (given that he does not have cystic fibrosis).

(c) Michael and Sally plan to have a child. Find the probability that the child will have cystic fibrosis (given that neither Michael nor Sally has it). (You may assume that Michael's genes are independent of Sally's.)