QUEEN MARY, UNIVERSITY OF LONDON

MAS 108 Probability I

Assignment 2 For handing in on 10 October 2005

Write your name and student number at the top of your assignment before handing it in. Staple all the pages together. Post the assignment in the red post-box on the ground floor of the Maths building before 1600 on Monday.

This week's reading: Devore, Chapter 2, Sections 2.2 and 2.3; *or* Hines and Montgomery, Chapter 2, Sections 2.5 and 2.6; *or* Rice, Chapter 1, Sections 1.3 and 1.4.

1 (20 marks) You throw an ordinary six-sided die twice and note the numbers showing, as in Assignment 1, Question 1. Assuming that all outcomes are equally likely, find the probability of each of the following events:

(a) a six turns up exactly once;

(b) both numbers are even;

(c) both numbers are even and exactly one of them is a six;

(d) both numbers are even or exactly one of them is a six.

2 (10 marks) Events A and B satisfy $P(A) = 1/2$, $P(B) = 9/20$ and $P(A \cap B) = 1/5$. Find the probabilities of

$$\text{(a) } A \cup B; \quad \text{(b) } B'; \quad \text{(c) } A \setminus B; \quad \text{(d) } A' \cap B'.$$

3 (10 marks) At a demonstration in Hyde Park, 50% of the demonstrators are long-haired, 45% are young and 20% are both young and long-haired. What are the probabilities that a person chosen at random among the demonstrators is (a) either young or long-haired (b) not young (c) long-haired but not young (d) neither young nor long-haired?

4 (20 marks) You go to the Students' Union to buy a piece of fruit. They have apples, oranges, bananas, pears and nectarines for sale. The probability that you buy a pear is three times the probability that you buy a nectarine; the probability that you buy an apple is twice the probability that you buy a nectarine; the probability that you buy a banana is twice the probability that you buy an apple; the probabilities of buying an apple or an orange are equal. You are certain to buy exactly one piece of fruit. Find the probability that you buy a banana.

5 (20 marks) Prove that, if events A and B are disjoint then $P(A)+P(B) \leq 1$. (Each step in your argument should be justified by citing an axiom or a result from lectures.)

6 (20 marks) A bicycle manufacturer checks the quality of the factory's output by randomly sampling the bicycles which are made. Defects in a bicycle are classified according to the part of the bicycle containing them: the frame (e.g. the crossbar), moving parts (e.g. gear wheels), elsewhere (e.g. bell). Suppose that he samples just one bicycle. Let

$$\begin{aligned} F &= \text{"there is any defect in the frame"} \\ M &= \text{"there is any defect in a moving part"} \\ E &= \text{"there is any defect elsewhere in the bicycle".} \end{aligned}$$

Suppose also that 1% of bicycles have any defect in the frame, 2% have any defect in a moving part, 1.5% have any defect elsewhere, 0.5% have defects in the frame and in a moving part, 0.4% have defects in the frame and elsewhere, (where 'elsewhere' means 'other than in the frame or in a moving part'), 0.6% have defects in a moving part and elsewhere, and 0.2% have defects in all three areas.

Describe the following events in words and find their probabilities:

(a) $F \cap M'$;

(b) $F' \cup M'$;

(c) $F \cup M \cup E$.