QUEEN MARY, UNIVERSITY OF LONDON

MAS 108 Probability I

Assignment 10 **For handing in on 12 December 2005**

Write your name and student number at the top of your assignment before handing it in. Staple all the pages together. Post the assignment in the red post-box on the ground floor of the Maths building before 1600 on Monday.

This week's reading: Devore, Chapter 4, Section 4.3, and Chapter 5, Sections 5.1, 5.2 and 5.5; *or* Hines and Montgomery, Chapter 5, Sections 5.1–5.5, 5.7, 5.8 and 5.12, and Chapter 8, Section 8.5; *or* Rice, Chapter 3, Sections 3.1–3.2 and 3.4–3.5, and Chapter 4, Sections 4.1, 4.3 and 4.4, and Chapter 5, Example F.

1 (Continued from Assignment 6) (50 marks) A baby has fifteen wooden blocks in a basket: five are yellow, five are red and five are blue. She takes out three blocks at random and builds a tower with them. Assume that every set of three blocks is equally likely to be chosen. Let Y be the number of yellow blocks in the tower and let Z be the number of red blocks in the tower.

(a) Write down the joint probability mass function for Y and Z.

(b) Find the marginal distribution of Y. Hence calculate $E(Y)$ and $\mathrm{Var}(Y)$.

(c) Calculate

  (i) $\mathrm{Cov}(Y,Z)$;

  (ii) $\mathrm{corr}(Y,Z)$.

(d) Find

  (i) $E(Y-Z)$;

  (ii) $\mathrm{Var}(Y-Z)$.

(e) Are Y and Z independent of each other?

2 (20 marks) There are 208 students taking Probability I. On the first day of term, each of them goes to the Student Union and buys a piece of fruit. For each of them independently, the probability that they buy a pear is $1/4$.

Let X be the number of students taking Probability I who buy a pear.

(a) Find E(X).

(b) Find $P(50 \leq X \leq 60)$.

3 (30 marks) Complete the proof of Theorem 9 by proving that

(a) if a is constant then $\mathrm{Cov}(aX,Y) = a\,\mathrm{Cov}(X,Y)$;

(b) if b is constant then $\mathrm{Cov}(X,Y+b) = \mathrm{Cov}(X,Y)$.

There will be no classes on Thursday 15 December or Friday 16 December. Marked work may be collected in Basic Medical Sciences Room 328 between 1400 and 1500 on Thursday, or from my office between 1430 and 1530 on Friday.

Challenge Problem *This question is not compulsory, and you probably cannot do it in time for the hand-in date. However, anyone who does it has a chance to improve their worst coursework mark so far. Put your solution in the red post-box on the ground floor of the Maths building before 1600 on Friday 16 December 2005. Collect marked solutions from me in my office hours next semester.*

The Students' Union Shop sells breakfast cereal made by two manufacturers.

(a) Manufacturer A includes signed photographs of Professor Bailey and Professor Tavakol in equal proportions. Each packet contains one photograph, and photographs are put in packets at random. Let N be the number of packets that you have to buy to get both photographs. Find $E(N)$.

(b) Manufacturer B includes signed photographs of Dr. Nelson and Dr. Soicher. Each packet contains one photograph, and photographs are put in packets at random, in such a way that the probability of getting a photograph of Dr. Nelson in any packet is $1/10$. Let M be the number of packets that you have to buy to get both photographs. Find $E(M)$.

(c) What lesson do you learn for cereal manufacturers?