

QUEEN MARY, UNIVERSITY OF LONDON

MAS 417

MTHM028

Association Schemes and Partially

Balanced Designs

Assignment 2

For handing in on 13 February 2006

- 1 Verify that C_1 in the Hamming scheme defines a distance-regular graph.
- 2 Draw a connected regular graph that is not distance-regular.
- 3 For $\mathbb{Z}_6$, write down the circulant matrices M_1 and M_3 .
- 4 Verify that, for $\mathbb{Z}_n$, $M_\alpha M_\beta = M_{\alpha+\beta}$ and $M_\alpha \chi_\omega = \chi_{\omega-\alpha}$.
- 5 Verify that the sets $\{0\}$, $\{1, 4, -4, -1\}$, $\{2, 7, -7, -2\}$, $\{3, -3\}$, $\{5, -5\}$ and $\{6, -6\}$ form a blueprint for $\mathbb{Z}_{15}$.
- 6 Write down as many association schemes as you can think of which have 15 elements. Give enough detail about them to show that they are all distinct.
- 7 Let $\mathcal{A}$ be the Bose–Mesner algebra of the group-divisible association scheme $\text{GD}(b, k)$ on a set Ω of size bk , where $b \geq 2$ and $k \geq 2$. Let G be the adjacency matrix for the relation “is in the same group as” on Ω .
 - (a) Show that $\{I, G, J\}$ is a basis for $\mathcal{A}$.
 - (b) Write down the multiplication table for $\{I, G, J\}$.
 - (c) Find all the idempotents in $\mathcal{A}$, expressing them in terms of this basis.
- 8 Prove the two orthogonality relations for the D matrix.
- 9 Find the character table, and the D matrix, for the cube association scheme.
- 10 There is a distance-regular graph with the parameters shown below. Express A_2 and A_3 as polynomials in A_1 . Hence find the minimal polynomial of A_1 . Hence find the character table.

