

QUEEN MARY, UNIVERSITY OF LONDON

MTH 4106

Introduction to Statistics

Assignment 5

For handing in on 14–15 February 2012

You should attempt all of these questions, as they are designed to help you to learn and understand the material in the course.

The ‘Feedback’ question is the one for handing in. Write your name, student number and group number at the top of your answer before handing it in. Staple all the pages together. Either (i) hand it to your allocated tutor when (s)he asks for it during your allocated Minitab laboratory session on Tuesday 14 February 2012, or (ii) post it in the red post-box on the ground floor of the Maths building before 1400 on Wednesday 15 February 2012.

If you want help on any of the other questions, or want to check that you have done them correctly, you may ask any tutor during your laboratory session or ask me in any of my office hours.

Please do not use Minitab at Queen Mary on Tuesdays except in your allocated hour.

1 This question refers to the example about Pete’s Pizza in the lectures. Let p be the unknown proportion of Pete’s customers who like anchovy topping. The two hypotheses are $H_0 : p \geq 0.2$ and $H_1 : p < 0.2$.

Pete takes a random sample of 20 of his customers and records which topping they ask for. Let X be the number of these who ask for anchovy topping. Then $X \sim \text{Bin}(20, p)$. The table below shows some possible values of p and some possible rejection regions R . Make a copy of this, and fill it in so that the entry in row R and column p is $P(X \in R \mid \text{the true proportion is } p)$.

rejection region R	true value of p						
	0.05	0.10	0.15	0.20	0.25	0.30	0.35
$\{0\}$							
$\{0, 1\}$							
$\{0, 1, 2\}$							
$\{0, 1, 2, 3\}$							

Use these values to work out the entries in the second table (overleaf), which shows the probability of making an incorrect conclusion if the true proportion is p and the null hypothesis is rejected if $X \in R$.

rejection region R	true value of p						
	0.05	0.10	0.15	0.20	0.25	0.30	0.35
	$\beta(p)$	Type II		$\alpha(p)$	Type I		
$\{0\}$							
$\{0, 1\}$							
$\{0, 1, 2\}$							
$\{0, 1, 2, 3\}$							

2 Suppose that the null and alternative hypotheses are changed to $H_0 : p \geq 0.3$ and $H_1 : p < 0.3$, and that we want the probability of Type I error to be no more than 5%.

- Write down all the rejection regions of the form $\{0, \dots, m\}$ that have the property that the probability of Type I error is no more than 5%.
- Which of these rejection regions is best to use for the hypothesis test? Why?
- What is the significance level of the test if this rejection region is used?
- Pete asks his sample of 20 customers, and finds that exactly one of them asks for anchovy topping. Report the result of the hypothesis test in a precise statistical fashion, including the P -value.
- Then write down what you should tell Pete, who is not a statistician.

3 (Feedback) I keep a special coin which I toss whenever I have to decide something randomly. Someone has suggested to me that the coin may not be fair. I am going to toss it 20 times and record the number X of heads. Let $p = \mathbb{P}(\text{heads})$ on each toss.

- Write down the null and alternative hypotheses.
- Consider rejection regions of the form $\{0, 1, \dots, k\} \cup \{20 - k, \dots, 19, 20\}$ with $0 \leq k \leq 9$. Which of these have the property that the probability of Type I error is no more than 5%?
- Use Minitab to plot the power curves for the tests with $k = 4$, $k = 5$ and $k = 6$.
- Which of these rejection regions is best to use for the hypothesis test? Why?
- What is the significance level of the test if this rejection region is used?
- I toss the coin 20 times, and find that it falls heads exactly 14 times. As a statistician, what do you conclude?