QUEEN MARY AND WESTFIELD COLLEGE

MAS 417 Association Schemes and Partially Balanced Designs

Assignment 5 **For handing in on 3 April 2001**

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1 Consider the following incomplete-block designs for eight treatments in six blocks of size four.

(a) The treatments are the vertices of the cube. The blocks are the faces of the cube.

(b) The cyclic design generated by $\{0, 2, 4, 6\}$ and $\{0, 1, 4, 5\}$ modulo 8.

Show that each design is partially balanced. Find its canonical efficiency factors, and the variances of all simple contrasts. Which design do you think is better?

2 Consider the following incomplete-block designs for fifteen treatments in sixty blocks of size three.

(a) The treatments are all 2-subsets of a 6-set. The blocks are all triples like $\{\{1, 2\}, \{1, 3\}, \{2, 3\}\}$, each occurring 3 times.

(b) The treatments are all 2-subsets of a 6-set. The blocks are all triples like $\{\{1, 2\}, \{3, 4\}, \{5, 6\}\}$, each occurring 4 times.

(c) The cyclic design generated by $\{0, 5, 13\}$, $\{0, 1, 5\}$, $\{0, 3, 14\}$ and $\{0, 6, 8\}$ modulo 15.

Show that each design is partially balanced. Find its canonical efficiency factors, and the variances of all simple contrasts. Which design do you think is best?

3 Draw the Hasse diagram for each of the following sets $\mathcal{F}$ of partitions. Find the zeta function and Möbius function in each case. For the association scheme defined by each one, write down the associate classes in words.

(a) The set consists of bk elements, grouped into b blocks of size k, and $\mathcal{F} = \{E, \text{blocks}, U\}$.

(b) The set is an $m \times n$ rectangle, and $\mathcal{F} = \{E, \text{rows}, \text{columns}, U\}$.

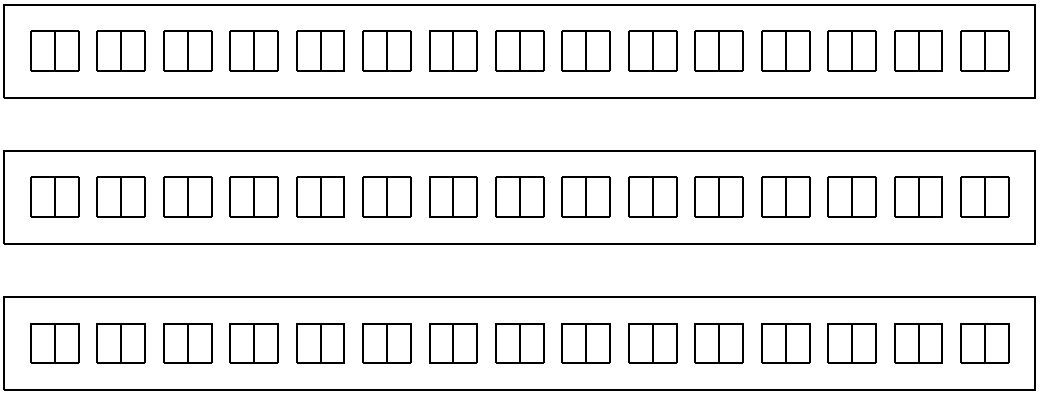

Figure 1: Three herds of 15 cows with 2 ears each

(c) In an experiment on types of ear-tag, the experimental units ("plots") are cows' ears. The experiment uses both ears from each of 15 cows from each of 3 herds. See Figure 1. The set consists of the 90 ears, and $\mathcal{F} = \{E, \text{cows}, \text{herds}, U\}$.

(d) An orthogonal block structure of your own choice, different from those above.

4 The web page for this course is at `http://www.maths.qmw.ac.uk/~rab/MAS417/` and includes a colour picture of the cube association scheme. Choose another non-trivial association scheme with n points, where $n \leq 16$. Write the `html` code to show it as a $n \times n$ square with colours to indicate the associate classes. Email this to `r.a.bailey@qmw.ac.uk`. The best entries will be put on the web page.