QUEEN MARY AND WESTFIELD COLLEGE

MAS 417 Association Schemes and Partially Balanced Designs

Assignment 4 **For handing in on 20 March 2001**

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1 For each of the values of n, a, p and q below, either describe a strongly regular graph on n vertices with valency a such that every edge is contained in p triangles and every non-edge is contained in q paths of length 2, or prove that no such strongly regular graph exists.

(a) $n = 13, \quad a = 6, \quad p = 2, \quad q = 3.$

(b) $n = 13, \quad a = 6, \quad p = 5, \quad q = 0.$

(c) $n = 16, \quad a = 9, \quad p = 4, \quad q = 6.$

(d) $n = 16, \quad a = 9, \quad p = 6, \quad q = 3.$

(e) $n = 20, \quad a = 4, \quad p = 3, \quad q = 0.$

(f) $n = 20, \quad a = 6, \quad p = 1, \quad q = 2.$

(g) $n = 170, \quad a = 13, \quad p = 0, \quad q = 1.$

2 Construct a square lattice design for 25 treatments in 15 blocks of size 5.

3 For each of the following incomplete-block designs, decide whether it is partially balanced. If it is partially balanced, give the association scheme with the smallest number of classes with respect to which it is partially balanced. If it is not partially balanced, explain why it is not.

(a) The treatment set is $\{a,b,\ldots,n\}$ and the blocks are $\{a,b,d,h,i,k\}$, $\{b,c,e,i,j,l\}$, $\{c,d,f,j,k,m\}$, $\{d,e,g,k,l,n\}$, $\{e,f,a,l,m,h\}$, $\{f,g,b,m,n,i\}$, $\{g,a,c,n,h,j\}$.

(b) The treatment set is $\{1,2,3,4,5,6,7\}$ and the blocks are $\{1,2,3\}$, $\{1,4,5\}$, $\{1,6,7\}$, $\{2,3,7\}$, $\{2,4,6\}$, $\{3,5,6\}$, $\{4,5,7\}$.

(c) The treatment set is $\{1,2,3,4,5,6\}$ and the blocks are $\{1,2,4\}$, $\{2,3,5\}$, $\{3,4,1\}$, $\{4,5,2\}$, $\{5,1,3\}$, $\{1,2,6\}$, $\{2,3,6\}$, $\{3,4,6\}$, $\{4,5,6\}$, $\{5,1,6\}$.

(d) The treatment set is $\{0,1,2,3,4,5,6,7,8,9\}$ and the blocks are $\{0,1\}$, $\{1,2\}$, $\{2,3\}$, $\{3,4\}$, $\{4,5\}$, $\{0,5\}$, $\{0,6\}$, $\{0,7\}$, $\{0,8\}$, $\{0,9\}$, $\{1,6\}$, $\{1,7\}$, $\{1,8\}$, $\{1,9\}$, $\{2,6\}$, $\{2,7\}$, $\{2,8\}$, $\{2,9\}$, $\{3,6\}$, $\{3,7\}$, $\{3,8\}$, $\{3,9\}$, $\{4,6\}$, $\{4,7\}$, $\{4,8\}$, $\{4,9\}$, $\{5,6\}$, $\{5,7\}$, $\{5,8\}$, $\{5,9\}$.

(e) The treatment set is $\{a,b,\ldots,l\}$ and the blocks are $\{a,b,c\}$, $\{d,e,f\}$, $\{g,h,i\}$, $\{j,k,l\}$, $\{d,g,j\}$, $\{a,h,k\}$, $\{b,e,l\}$, $\{c,f,i\}$, $\{e,i,k\}$, $\{a,f,j\}$, $\{b,d,h\}$, $\{c,g,l\}$.

(f) The treatment set is $\{1,2,3,4,5,6,7,8\}$ and the blocks are $\{1,5,8\}$, $\{2,6,1\}$, $\{3,7,2\}$, $\{4,8,3\}$, $\{5,1,4\}$, $\{6,2,5\}$, $\{7,3,6\}$, $\{8,4,7\}$.

4 Let N be the incidence matrix for an incomplete-block design with block set Δ and treatment set Θ. The *complement* of this design has incidence matrix N^*, where $N^*(\delta,\theta) = 1 - N(\delta,\theta)$. Prove that the complement of a partially balanced incomplete-block design is partially balanced with respect to the same association scheme.