

QUEEN MARY AND WESTFIELD COLLEGE

MAS 417

Association Schemes and Partially Balanced Designs

Assignment 2

For handing in on 20 February 2001

- 1 Find the parameters of the Johnson scheme $J(7, 3)$.
- 2 Find the parameters of the Hamming scheme $H(5, 4)$.
- 3 Verify that C_1 in the Johnson scheme defines a distance-regular graph.
- 4 Draw a connected regular graph that is not distance-regular.
- 5 For \mathbf{Z}_6 , write down the circulant matrices M_1 and M_3 .
- 6 Verify that, for \mathbf{Z}_n , $M_\alpha M_\beta = M_{\alpha+\beta}$ and $M_\alpha \chi_\omega = \chi_{\omega-\alpha}$.
- 7 Verify that the sets $\{0\}$, $\{1, 4, -4, -1\}$, $\{2, 7, -7, -2\}$, $\{3, -3\}$, $\{5, -5\}$ and $\{6, -6\}$ form a blueprint for \mathbf{Z}_{15} .
- 8 Write down as many association schemes as you can think of which have 15 elements. Give enough detail about them to show that they are all distinct.
- 9 Let \mathcal{A} be the Bose-Mesner algebra of the group divisible association scheme $\text{GD}(b, k)$ on a set Ω of size bk . Let G be the adjacency matrix for the relation “is in the same group as” on Ω .
 - (a) Show that $\{I, G, J\}$ is a basis for \mathcal{A} .
 - (b) Write down the multiplication table for $\{I, G, J\}$.
 - (c) Find all the idempotents in \mathcal{A} , expressing them in terms of this basis.