QUEEN MARY, UNIVERSITY OF LONDON

MAS 305 — Algebraic Structures II

Key Objectives — Autumn 2006

Groups and Rings: basics Know the axioms and be able to prove elementary consequences. Be able describe subgroups, normal subgroups, subrings and ideals, and to test whether a given subset is one of those. Be familiar with some standard examples, such as cyclic, symmetric, alternating and dihedral groups, matrix groups in dimension 2 over a field, the ring of integers, the integers modulo n, polynomial rings and matrix rings.

Groups and Rings: quotients, and the isomorphism theorems Know the construction of quotient rings and quotient groups, and be familiar with its properties. Be able to calculate in quotient groups or quotient rings by using coset representatives. Be able to state, prove and use the three isomorphism theorems and the Correspondence Theorem.

Group actions, including conjugacy Be able to test when a mapping is a group action. Know the definition of kernel, orbit, stabilizer, conjugate, centralizer, normalizer, centre. Be able to state, prove and use Cayley's Theorem, Cauchy's Theorem, the Orbit-Stabilizer Theorem, the theorem attributed to Burnside, and the theorem that if a group has a subgroup of index n then it has a transitive subgroup of S_n as a quotient. Be familiar with applications to the structure of finite groups of prime-power order.

Sylow's theorems for groups Be able to state and prove Sylow's three theorems, and apply them to investigate the structure of small finite groups.

Simple groups and composition series Know the definition of simple group and the facts that alternating groups are simple but the only simple groups of prime-power order are cyclic of prime order. Be able to define a composition series and a soluble group. Be able to state, prove and use the Jordan-Hölder Theorem and basic theorems about finite soluble groups.

Simple rings Know, and be able to prove, that matrix rings over a field are simple.

Factorization in integral domains Know what units, associates, irreducibles and highest common factors are. Know what is a unique factorization domain, a principal ideal domain, and a Noetherian integral domain; know, and be able to prove, the implications between them.

Noetherian rings Know the definition of Noetherian ring. Be able to state, prove and use the two conditions equivalent to being Noetherian, Hilbert's Basis Theorem, and other properties of Noetherian rings.

Direct sums and products Know the definition and simple properties of the direct product of groups and the direct sum of rings, and be able to use these to construct examples.

The examination will contain at least SIX questions. The rubric will be as follows. Note that this is the same as last year's rubric but slightly different from the rubric in previous years.

> *You may attempt as many questions as you wish and all questions carry equal marks. Except for the award of a bare pass, only the best FOUR questions answered will be counted.*
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> *Calculators are NOT permitted in this examination. The unauthorized use of a calculator constitutes an examination offence.*

R. A. Bailey
3 October 2006