QUEEN MARY, UNIVERSITY OF LONDON

MAS 305 Algebraic Structures II

Assignment 8 **For handing in on 13 December 2006**

Write your name and student number at the top of your assignment before handing it in. Staple all the pages together. Give the completed assignment to me in person in the Wednesday lecture.

1 Let I be the ideal $\langle\{2,x^2\}\rangle$ in $\mathbb{Z}[x]$. Calculate the chain of ideals $L_n(I)$ for $n \geq 0$.

2 Let J be the ideal $\langle\{8,2+3x\}\rangle$ in $\mathbb{Z}[x]$. Calculate the chain of ideals $L_n(J)$ for $n \geq 0$.

3 Let J be the principal ideal $\langle 1+3x+2x^2\rangle$ in $\mathbb{Z}_6[x]$. Calculate the chain of ideals $L_n(J)$ for $n \geq 0$.

4 Let R be an integral domain. Let $f(x) = a_0 + a_1x + \cdots + a_kx^k$ in $R[x]$, where $k \geq 0$ and $a_k \neq 0$, and let J be the principal ideal $\langle f(x)\rangle$. Calculate the chain of ideals $L_n(J)$ for $n \geq 0$.

Hence prove that the ideal in Question 1 is not a principal ideal.

5 Let R be a commutative ring with identity and let I be an ideal in R. Prove that $I[x]$ is an ideal of $R[x]$.

Hence or otherwise, prove that if the polynomial ring $R[x]$ is Noetherian then R is Noetherian.

6 The following rings are all Noetherian. For each one give a very brief proof using results from lectures or other information about their ideals.

(a) $\mathbb{C}$

(b) $\mathbb{Z}$

(c) $\mathbb{Z}_6[x,y]$

(d) $\left\{a+b\sqrt{3} : a,b \in \mathbb{Z}\right\}$

(e) $M_3(\mathbb{R})$