QUEEN MARY, UNIVERSITY OF LONDON

MAS 305 Algebraic Structures II

Assignment 7 For handing in on 6 December 2006

Write your name and student number at the top of your assignment before handing it in. Staple all the pages together. Give the completed assignment to me in person in the Wednesday lecture.

This week's reading: Cameron, Sections 2.2, 2.3, 7.2.1 and 7.2.2 *or* Hartley and Hawkes, Sections 2.2 and 4.1–4.4.

1 Let R_1, R_2 and R_3 be rings, and let $\phi_1 : R_1 \to R_2$ and $\phi_2 : R_2 \to R_3$ be ring homomorphisms. Prove that $\phi_1\phi_2$ is a homomorphism from R_1 to R_3.

2 Prove the Second Isomorphism Theorem for rings.

3 Prove the Third Isomorphism Theorem for rings.

4 Let R be a commutative ring with identity. Show that R is simple if and only if R is a field.

5 Find an ideal of $M_2(2\mathbb{Z})$ which is not equal to $M_2(I)$ for any ideal I of $2\mathbb{Z}$.

6 Let r and s be elements of an integral domain R. Prove that if r is irreducible and s is an associate of r then s is irreducible.

7 Let $R = \left\{a + b\sqrt{-5} : a, b \in \mathbb{Z}\right\}$.

(a) Show that R is a subring of $\mathbb{C}$.

(b) Find all the units in R.

(c) Prove that R is not a unique factorization domain.