QUEEN MARY, UNIVERSITY OF LONDON

MAS 305 Algebraic Structures II

Assignment 6 For handing in on 22 November 2006

Write your name and student number at the top of your assignment before handing it in. Staple all the pages together. Give the completed assignment to me in person in the Wednesday lecture.

This week's reading on groups: Cameron, Sections 7.1.3 and 7.1.4; *or* I. Stewart, *Galois Theory*, Chapter 13; *or* J. S. Rose, *A Course on Group Theory*, Chapter 7.

This week's reading on rings: Cameron, Section 2.1; *or* R. Hartley and T. O. Hawkes, *Rings, Modules and Linear Algebra*, Sections 1.1–2.1.

1 Prove that $\mathrm{GL}(2,3)$ is soluble.

2 Prove that if all the Sylow subgroups of a finite group G are normal in G then G is soluble.

3 Let R be a ring. Prove that $a0_R = 0_R$ for all a in R.

4 Find a zero-divisor in $M_2(\mathbb{Z})$.

5 Find all the units in the rings (a) $\mathbb{Z}_8$ (b) $\mathbb{Z}_{18}$. Show that one of these groups of units is cyclic and the other is not.

6 Let S be the set of all elements of $M_2(\mathbb{Z})$ of the form $\begin{bmatrix} a & b \\ 0 & c \end{bmatrix}$ for a, b, c in $\mathbb{Z}$. Is S a subring of $M_2(\mathbb{Z})$? Is S an ideal of $M_2(\mathbb{Z})$?

7 Prove that if $I_1, \ldots, I_n$ are ideals of a ring R, for some positive integer n, and $J = I_1 \cap I_2 \cap \cdots \cap I_n$, then J is an ideal of R.