QUEEN MARY, UNIVERSITY OF LONDON

MAS 305 — Algebraic Structures II

Assignment 3 — For handing in on 25 October 2006

Write your name and student number at the top of your assignment before handing it in. Staple all the pages together. Give the completed assignment to me in person in the Wednesday lecture.

This week's reading: Cameron, Sections 3.3.2, 3.3.3, 3.4.1 and 7.1.1.
Ledermann and Weir, Chapter 5 and Sections 3.3.1 and 3.3.2.

1 Let $T = \{3^n : n \in \mathbb{Z}\}$.

(a) Prove that T is a normal subgroup of $(\mathbb{R} \setminus \{0\}, \times)$.

(b) Let K be the set of all elements of $\mathrm{GL}(2,\mathbb{R})$ whose determinant is in T. Prove that K is a normal subgroup of $\mathrm{GL}(2,\mathbb{R})$.

2 Let G be the group of all 3-dimensional rotational symmetries of the cube. We know that $|G| = 24$.

(a) Find the order of the stabilizer of a vertex.

(b) Find the order of the stabilizer of an edge.

(c) Hence or otherwise, write down all the elements of G, either in words or as suitable permutations.

3 Prove that $\mathrm{GL}(2,2) \cong S_3$.

4 Let G be a finite group which acts transitively on a set Ω of size $n \geq 2$. Prove that there is some element g in G with $f(g) = 0$, where $f(g)$ is defined as in the theorem wrongly attributed to Burnside.

5 (a) Find the conjugacy classes in D_{10}.

(b) For one element in each conjugacy class, find its centralizer.

(c) Find all normal subgroups of D_{10}.