QUEEN MARY, UNIVERSITY OF LONDON

MAS 305 — Algebraic Structures II

Assignment 2 — **For handing in on 11 October 2006**

Write your name and student number at the top of your assignment before handing it in. Staple all the pages together. Give the completed assignment to me in person in the Wednesday lecture.

This week's reading: Cameron, Sections 3.3 and 3.4.

1 Let ϕ be a homomorphism from a group G to a group H.

(a) Prove that $1_G\phi = 1_H$.

(b) Prove that $\left(g^{-1}\right)\phi = (g\phi)^{-1}$ for all g in G.

(c) If ϕ is a bijection then it has an inverse bijection ϕ^{-1} from H to G. Prove that ϕ^{-1} is also a homomorphism.

2 Are the following homomorphisms? In each case, either give a proof or give a reason why not.

(a) G is Abelian, $H = G$, and $g\phi = g^2$ for g in G.

(b) $G = D_{10}$, $H = G$, and $g\phi = g^2$ for g in G.

(c) $G = D_{10}$, $H = \{1, -1\}$ under multiplication, and $g\phi = \begin{cases} 1 & \text{if } g \text{ is a rotation} \\ -1 & \text{if } g \text{ is a reflection} \end{cases}$.

3 Let G_1, G_2 and G_3 be groups, and let $\phi\colon G_1 \to G_2$ and $\theta\colon G_2 \to G_3$ be homomorphisms. Prove that $\phi\theta$ is a homomorphism from G_1 to G_3.

4 Let G and H be groups, and let $\phi\colon G \to H$ be a homomorphism. Prove that if G is cyclic then $\mathrm{Im}(\phi)$ is cyclic.

5 Let G be a finite group in which every element g satisfies $g^2 = 1_G$.

(a) Prove that G is Abelian.

(b) Show that either $|G| = 1$ or G has a normal subgroup K of order 2.

(c) Hence prove that $|G|$ is a power of 2.

6 Write down all the elements of S_3 in columns according to their cycle structure.

Using theorems and a minimum of brute force, find all subgroups of S_3, and say which of them are normal.

7 Repeat the previous question with A_4 in place of S_3.