QUEEN MARY, UNIVERSITY OF LONDON

MAS 305 — Algebraic Structures II

Assignment 1 — For handing in on 4 October 2006

Write your name and student number at the top of your assignment before handing it in. Staple all the pages together. Give the completed assignment to me in person in the Wednesday lecture.

This week's reading: Cameron, Sections 1.2, 3.1 and 3.2.

1 Let G be the group of all symmetries of the regular pentagon.

(a) Write down all the elements of G. (You will probably need to draw a picture to explain this.)

(b) Is G Abelian?

(c) Is G cyclic?

(d) Does G have a subgroup of order 4?

(e) Does G have a subgroup of order 5?

(f) Does G have a subgroup of order 6?

(Note that *Yes* and *No* are almost never sufficient for an answer: you need to give reasons.)

2 Prove that if $H_1, \ldots, H_n$ are subgroups of a group G, for some positive integer n, and $K = H_1 \cap H_2 \cap \cdots \cap H_n$, then K is a subgroup of G.

Would your proof work for an infinite collection of subgroups of G?

3 Let H be the set of all elements of $\mathrm{GL}(2,\mathbb{R})$ of the form $\begin{bmatrix} 1 & n \\ 0 & 1 \end{bmatrix}$ for $n \in \mathbb{Z}$, and let K be the set of all elements of $\mathrm{GL}(2,\mathbb{R})$ with determinant 1. Prove that H and K are both subgroups of $\mathrm{GL}(2,\mathbb{R})$.

Is either of them Abelian?

Is either of them normal in $\mathrm{GL}(2,\mathbb{R})$?

4 Prove that the relation $\sim_L$ is an equivalence relation.

5 Consider the group S_4 of all permutations of $\{1,2,3,4\}$. Put

$$H = \{g \in S_4 : 3g = 3\}.$$

(a) Without writing down its elements, prove that H is a subgroup of S_4.

(b) How many cosets does H have in S_4?

(c) Let x be the permuation $(1\,3\,4)$. Prove that $y \in Hx$ if and only if $3y = 4$.

(d) Is Hx also a left coset of H?

6 Let G be the group of positive real numbers under multiplication.

(a) Prove that, if $x, y \in G$ and $x > y > 1$ then $y \notin \langle x \rangle$.

(b) Prove that G is not cyclic.

7 Let G be a finite group of order p, where p is prime. Prove that G is cyclic.

8 What is the order of $\mathrm{GL}(2,2)$? Find all the subgroups of $\mathrm{GL}(2,2)$.