Orbital Tutte Polynomials

Jason Rudd

6th March 2006

The *Tutte polynomial*

$$T(\Gamma; x, y) = \sum_{D \subseteq E(\Gamma)} (x-1)^{\rho(E)-\rho(D)} (y-1)^{|D|-\rho(D)}$$

of a graph Γ can be used to count many structures on Γ, including nowhere zero flows, nowhere zero tensions, proper vertex colourings and acyclic orientations.

Given a graph Γ and a group $G \leq \mathrm{Aut}(\Gamma)$ of automorphisms of Γ, we aim to define a polynomial which counts orbits of G on as many of the structures on Γ counted by $T(\Gamma; x, y)$ as possible, while itself specializing to the Tutte polynomial.

The main tool for doing this will be the *orbit counting lemma*,

$$\#\ \text{orbits} = \frac{1}{|G|} \sum_{g \in G} \mathrm{fix}(g)$$

so the objective is to count the numbers of the various structures on Γ which are fixed by each automorphism g.