

Local to Global Structure in Buildings

by Mark Ronan

Buildings originally arose in connection with the groups of Lie type, but they also admit a purely combinatorial definition. They are amalgamations of apartments, which can be positively curved (the spherical case), negatively curved, or flat (the affine case). Spherical buildings of dimension at least 2, and affine buildings of dimension at least 3, exhibit large automorphism groups, and in the spherical case, though not in the affine case, the local structure determines the global structure. I shall explain why. For hyperbolic buildings of dimension at least 3 it is not known whether the local structure determines the global structure, nor whether there is a large automorphism group, but a good way to approach the problem is to use 'twin buildings'.

Twin buildings first arose from Kac-Moody groups, and are analogous to spherical buildings. In most cases their local structure determines their global structure, and there is a large automorphism group, so it becomes an intriguing question whether a single building has a twin. This talk will explain what is currently known. If the local structure determines the global structure then this would have implications for finite geometries that are locally isomorphic to hyperbolic buildings.