

Symmetry and the Monster

by Mark Ronan

This talk will describe the quest to find a complete list of all finite simple groups. These groups, the basic building blocks for all finite groups, were instrumental in Galois' work on the solvability of algebraic equations; and he himself discovered some important ones. More were exhibited in Jordan's great treatise in 1870, and further families emerged from Lie's work, as a result of the classification work by Killing and Cartan in the late nineteenth century.

After the Second World War new families of simple groups were discovered, and there was enormous interest in finding a complete list. A way forward was found using work of Richard Brauer, and the great theorem of Walter Feit and John Thompson. They showed that every finite simple group whose order is not a prime number must contain elements of order 2, leading to some important subgroups that offered a method for completing the list. To cut a long story short, while Thompson was advancing these new methods, Zvonimir Janko, a Croatian mathematician working in Australia, surprised the world with a very strange exceptional group. This was the first exception since Émile Mathieu discovered five beautiful groups of permutations in the nineteenth century, and it really set the cat among the pigeons.

Further new exceptions came thick and fast, and they were dubbed "sporadic groups". The largest is called the Monster. This talk will explain how the Monster was discovered, and how it came to reveal strange connections between number theory and mathematical physics, sometimes called the moonshine connections.