Cycling with mirrors, modulo n

Donald Preece

For an integer n, we wish to dispose the $n-1$ non-zero elements of $\mathbb{Z}_n$ in a circular arrangement $[a_1, a_2, ..., a_{n-1}]$ (with $a_n = a_1$) so that the set of differences $a_{i+1} - a_i$ (for $i = 1, 2, ..., n-1$) is itself $\mathbb{Z}_n \setminus \{0\}$. A basic recipe for doing this for any odd n was given in 1978 by Friedlander, Gordon and Miller. Variants of this recipe are available for $n \equiv 1 \pmod{2i}$, with $i = 2, 3, 4, \ldots$. Entirely different procedures have been published by the speaker for prime values of n . Now procedures have been found specifically for values $n = pq$ where p and q are distinct odd primes. Many of these new procedures produce arrangements in which any element x and its negative $-x \pmod{n}$ are $(n-1)/2$ positions apart (in either direction).

The talk will pay little heed to possible applications. However, for small values of n , the arrangements give us neighbour designs, as in statistical design theory.