

An introduction to proof mining

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In this talk we will look at some examples from mathematics where proofs carry more information than the theorem they prove. We will start with the simple proof that $\sqrt{2}$ is irrational, then look at Erdős's proof that there are infinitely many prime numbers, and some proofs involving the PHP. Finally we will look at Cheney's proof of uniqueness for L_1 approximation. I will show how each proof actually has some extra information (e.g. algorithm or bound) enough to derive a stronger theorem. The difficulty in obtaining the extra information normally lies in the use of "non-computable" proof techniques such as compactness arguments or proof by contradiction. Proof mining aims to identify and extract extra information from (non-computable) proofs in order to derive stronger (new) theorems. Proof mining can also be viewed as a methodology for deriving results in "hard mathematics" from corresponding results in "soft mathematics", using Terence Tao's terminology. A brief introduction to the proof-theoretic techniques involved will be surveyed, time permitting.