Defining $\mathbb{Z}$ in $\mathbb{Q}$

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Hilbert's 10th Problem was to find an algorithm that decides solvability of diophantine equations over the integers. Matyiasevich showed in 1970 that no such algorithm exists. The same problem over the rationals is still open. If one could find an existential first-order definition of $\mathbb{Z}$ in $\mathbb{Q}$ one would, again, obtain a negative solution. But whether this can be done is open as well. Instead, we will present a universal definition of $\mathbb{Z}$ in $\mathbb{Q}$, and prove model theoretically that - under a very mild arithmo-geometric conjecture (much weaker than Mazur's) - there is no existential definition of $\mathbb{Z}$ in $\mathbb{Q}$.