

INTRODUCTION TO EXCEPTIONAL LIE ALGEBRAS: THE COMPACT REAL FORM OF G_2

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The best-known examples of Lie algebras are matrix algebras with the Lie bracket $[A, B] = AB - BA$. For example the 2×2 matrices of trace 0 form a 3-dimensional Lie algebra, called A_1 . It is an easy exercise to find a basis for this (complex) algebra so that the Lie multiplication is given by the rule for the vector cross product.

There are four infinite families of complex Lie algebras A_n , B_n , C_n and D_n , and five exceptional algebras G_2 , F_4 , E_6 , E_7 and E_8 . I shall describe a (possibly new) construction of the 14-dimensional algebra G_2 which is more symmetrical than the usual construction.