

Deformations of the Modular Group

The modular group $PSL(2, Z)$ is the group of all Möbius transformations of the form

$$z \rightarrow \frac{az + b}{cz + d} \quad a, b, c, d \in Z \quad ad - bc = 1$$

acting on the Riemann sphere. As an abstract group it is the free product of a cyclic group of order 2 and a cyclic group of order 3. As a Kleinian group it has limit set the real axis union infinity. It is rigid as a Kleinian group with connected limit set, but it has a natural complex one-dimensional space of deformations as a holomorphic correspondence. I will show some pictures of these deformations, discuss the critical relations and boundary degeneracies that occur, and explain how their classification reduces to a combinatorial classification of ‘patterns on triangular tiles’.