# GROUP ACTIONS ON AMORPHOUS SETS

PETER J. CAMERON AND SAM TARZI

**Abstract.** A set $U$ is amorphous if every subset of $U$ is finite or cofinite; an amorphous set is *bounded* if there is a natural number $n$ such that every non-trivial partition of $u$ has all but finitely many parts of size $m$, for some $m \leq n$. Our main result asserts that, if $U$ is bounded amorphous, then the symmetric group on $U$ modulo the group of finitary permutations is isomorphic to a finite group, and that every finite group can occur in this situation. We also obtain some results about the structure of the finitary symmetric group on an amorphous set. Our results depend heavily on the classification of bounded amorphous sets by John Truss.

**§1. Permutations of an amorphous set.** A set is said to be *amorphous* if it is infinite but is not the disjoint union of two infinite sets. Amorphous sets are *Dedekind finite*, that is have no countably infinite subset. The presence of Dedekind finite cardinals means that the Axiom of Choice (AC) cannot hold. A set is *strictly amorphous* if in addition to being amorphous, it is also the case that in any partition of the set into infinitely many pieces, all except finitely many are singletons.

Truss and co-workers [3, 4, 6] have carried out an extensive study aimed at classification of amorphous sets and their neighbours by examining the structure that an amorphous set can carry. This work corroborated the idea that 'strictly amorphous' gives the smallest notion of infinite set. Truss used finite permutation groups as invariants to ascertain deviation from strict amorphousness. We will show that these groups have another role to play.

The *symmetric group* $\mathrm{Sym}(U)$ on a set $U$ consists of all permutations of $U$. A permutation is *finitary* if it fixes all but finitely many points of $U$. The finitary permutations of $U$ form a group $\mathrm{FSym}(U)$, the *finitary symmetric group* on $U$, which is a normal subgroup of $\mathrm{Sym}(U)$. A finitary permutation has a *parity* (even or odd), just as for a permutation of a finite set; the even permutations form a subgroup of index 2 in $\mathrm{FSym}(U)$, the *alternating group* $\mathrm{Alt}(U)$.

In ZFC, the factor group $\mathrm{Sym}(U)/\mathrm{FSym}(U)$ has cardinality $2^{|U|}$, and its normal subgroups form a chain. Indeed, the proper normal subgroups of $\mathrm{Sym}(U)$ are as follows:

- the trivial group;
- the alternating group;
- the bounded symmetric group $\mathrm{BSym}_\alpha(U)$ consisting of all permutations moving fewer than $\alpha$ points, for each infinite cardinal $\alpha \leq |U|$;
- $\mathrm{Sym}(U)$.

As we will see, the position is very different in the absence of the Axiom of Choice!

A *permutation group* on $U$ is a subgroup of $\mathrm{Sym}(U)$. It is *transitive* if the only fixed subsets of $U$ are $\emptyset$ and $U$, and *primitive* if in addition the only fixed partitions of $U$ are $\{U\}$ and the partition into singletons. See [1] for the terminology and results of permutation group theory.

THEOREM 1.1 (Jordan–Wielandt Theorem). *Let $U$ be an infinite set. Then a primitive permutation group on $U$ which contains a non-identity finitary permutation must contain the alternating group* $\mathrm{Alt}(U)$.

We remark that the proof of this theorem (see [1, p. 166]) makes no use of the Axiom of Choice, and is valid in ZF.

With the help of this theorem, all transitive permutation groups on a strictly amorphous set can be described. We begin with a general result which applies to any amorphous set.

LEMMA 1.2. *let $U$ be an amorphous set, and $g$ a permutation of $U$. Then all cycles of $g$ are finite; and there is an integer $n$ such that all but finitely many cycles have length $n$.*

PROOF. Suppose that $g$ has an infinite cycle. By replacing $g$ by $g^2$ if necessary, we can assume that $g$ has at least two infinite cycles; the point set of one of these cycles is then infinite and coinfinite.

Now, for every $n$, the set of points lying in cycles of length $n$ of $g$ must be finite or cofinite. If it is cofinite for some $n$, the conclusion of the lemma holds; so suppose not. Then there must be infinitely many different cycle lengths. Partition the set of cycle lengths into two infinite parts $A$ and $B$; then the set of points lying in cycles with lengths in $A$ is infinite and coinfinite. $\dashv$

COROLLARY 1.3. *For any amorphous set $U$, the group* $\mathrm{Sym}(U)$ *is a torsion group.*

PROOF. The preceding proof shows that, for any permutation $g$ of $U$, there is a number $n$ such that $g^n \in \mathrm{FSym}(U)$; and $\mathrm{FSym}(U)$ is clearly a torsion group. ⊣

We will draw a stronger conclusion for bounded amorphous sets in the next section.

THEOREM 1.4. *Let $U$ be a strictly amorphous set. Then*
1. $\mathrm{Sym}(U) = \mathrm{FSym}(U)$*;*
2. *the only transitive subgroups of* $\mathrm{Sym}(U)$ *are* $\mathrm{Sym}(U)$ *and* $\mathrm{Alt}(U)$.

PROOF. According to the lemma, each permutation has the property that all but finitely many points lie in cycles of length $n$, for some $n$. Now the condition that $U$ is strictly amorphous means that $n = 1$, so that $g$ is finitary.

If $G$ is transitive, then any invariant partition has all its parts of the same size, and so (since $U$ is strictly amorphous) must be trivial. The result now follows from the Jordan–Wielandt theorem. ⊣

**§2. Bounded amorphous sets.** An example will illustrate what can happen when we try to extend this result to bounded amorphous sets. Suppose first that the bound is 2, so that there is a partition of $U$ with all but finitely many points lying in parts of size 2. Now there is a permutation of $U$ which interchanges the points in each part of size 2 and fixes all other points. (No choice principle is required for this.) This element lies in $\mathrm{Sym}(U)$ but not in $\mathrm{FSym}(U)$, so that the factor group has order at least 2 (and it is easy to see that equality holds).

Next suppose that the bound is 3. There are two possible cycles on a set of size 3, corresponding to its two cyclic orientations. If there is a choice function for the set of orientations, we can use it to construct a permutation in which all but finitely many points lie in 3-cycles, so that $|\mathrm{Sym}(U)/\mathrm{FSym}(U)| = 3$. But if no such choice function exists, then $\mathrm{Sym}(U) = \mathrm{FSym}(U)$.

Now we turn to the general case. Let $U$ be an amorphous set. We consider a partition of $U$ to be *finitary* if all its parts are finite. If $\pi$ is a finitary partition of $U$, there is a unique integer $n(\pi)$ such that all but finitely many parts of $\pi$ have size $n(\pi)$ (arguing as we did for cycles of a permutation). The number $n(\pi)$ is the *gauge* of $\pi$. We say that $U$ is *bounded* or *unbounded* according as the gauges of its finitary partitions are. If $U$ is bounded, the *gauge* of $U$ is the largest gauge of a non-trivial partition of $U$.

The set $\Pi(U)$ of finitary partitions of $U$ is partially ordered by *refinement*: $\pi_1 \leq \pi_2$ if every member of $\pi_2$ is a union of members of $\pi_1$. The partitions form a complete lattice with respect to this order. Two partitions $\pi_1$ and $\pi_2$ are said to be *equivalent*, written $\pi_1 \sim \pi_2$, if one can be obtained from the other by finite alterations, that is, if there is an infinite set $X \subseteq U$ such that $X$ is a union of members of $\pi_1$ and for $\xi \subseteq X$, $\xi \in \pi_1 \Leftrightarrow \xi \in \pi_2$. A finitary partition $\pi$ is said to be in *standard form* if as many as possible of the members of $U$ are in members of $\pi$ of size $n(\pi)$, and any left over are in singletons of $\pi$; the union of the left over members of $\pi$ has cardinality less than the gauge of $\pi$. A member $\xi$ of a finitary partition $\pi \in U$ is called *atypical* if $|\xi| \neq n(\pi)$.

The following two properties of an amorphous set are proved in [6]:

(a) If $U$ is amorphous then $\Pi(U)$ is a lattice, and equivalence is a congruence on the lattice.

(b) For any $\pi \in \Pi(U)$, there exists $\pi_1 \in \Pi(U)$ in standard form equivalent to $\pi$. Moreover if $\pi_2$ is also in standard form, and has the same gauge as $\pi_1$, then $\pi_1$ and $\pi_2$ have an equal number of atypical elements.

For an amorphous set $U$ define $\Pi_n(U) := \{\pi \in \Pi(U) : n(\pi) = n\}$ and $N(U) := \{n : \Pi_n(U) \neq \emptyset\}$. It follows from (b) that there is a well-defined *excess function* $E_U$ of the amorphous set $U$, defined on $N(U)$ and given by the rule that $E_U(1) = 0$ and $E_U(n)$ is the number of singletons of a standard form member of $\pi \in \Pi_n(U)$ if $n > 1$.

For any bounded amorphous set $U$, there is $\pi \in \Pi(U)$ such that $n(\pi) = n(U)$. Such a partition is called *maximal*. The maximal partitions form a single equivalence class which is the greatest element of the lattice. The value of $E_U(n(U))$ is the *excess* of $U$.

PROPOSITION 2.1. *Let $U$ be a bounded amorphous set with gauge $n$. Then $\mathrm{Sym}(U)/\mathrm{FSym}(U)$ is a finite group with order dividing $n$.*

PROOF. First we show that $\mathrm{Sym}(U)/\mathrm{FSym}(U)$ is finite, of order at most $n!$. Suppose not, and let $g_1, \ldots, g_m$ be elements of $\mathrm{Sym}(U)$ in distinct cosets of $\mathrm{FSym}(U)$, with $m > n!$. Let $\pi$ be a maximal partition. Then all of $g_1, \ldots, g_m$ fix the equivalence class of $\pi$, so after altering them on a finite subset we can assume that they all fix $\pi$. Since the set of parts of $\pi$ is strictly amorphous, these permutations fix all but finitely many parts of $\pi$. For any $i \neq j$, the permutations $g_i$

and $g_j$ agree on either finitely or cofinitely many parts of $\pi$; the latter cannot be the case, or $g_i g_j^{-1} \in \mathrm{FSym}(U)$, contrary to assumption. So there are infinitely parts on which the permutations mutually disagree; this is impossible since there are more than $n!$ of them.

Now choose a part of $\pi$ fixed by a set of coset representatives of $\mathrm{FSym}(U)$ in $\mathrm{Sym}(U)$, and number its points $1, \ldots, n$. The map that takes an element of $\mathrm{Sym}(U)$ first to its coset representative and then to the induced permutation in $\mathrm{Sym}(n)$ is a homomorphism, with kernel $\mathrm{FSym}(U)$. It is clear that the homomorphism is independent of the choice of coset representatives; renumbering the points of the chosen part of $\pi$ merely replaces the image by a conjugate in $\mathrm{Sym}(n)$. Since each element of $\mathrm{Sym}(U)$ has all but finitely many of its cycles of the same length, we see that the image of this homomorphism consists of permutations with all cycles of the same length. So the image is semiregular, and has order dividing $n$. The proof is complete. $\dashv$

Corollary 2.2. *If $U$ is a bounded amorphous set, then* $\mathrm{Sym}(U)$ *is locally finite.*

Proof. This is immediate from the fact that it has a locally finite normal subgroup $\mathrm{FSym}(U)$ of finite index. $\dashv$

Proposition 2.3. *For an amorphous set $U$, there is an order-preserving injection from the lattice of subgroups of* $\mathrm{Sym}(U)/\mathrm{FSym}(U)$ *to the lattice of equivalence classes of partitions of $U$.*

Proof. Let $H$ be a subgroup of $\mathrm{Sym}(U)/\mathrm{FSym}(U)$. As shown above, if we choose coset representatives $g_1, \ldots, g_r$ for $\mathrm{FSym}(U)$ in $\mathrm{Sym}(U)$, there is a maximal partition $\pi$ fixed by $\langle g_1, \ldots, g_r \rangle = G'$, and $G'$ acts semiregularly on all but finitely many parts of $\pi$. Now the group $H'$ generated by the coset representatives in $H$ has orbits of size $|H|$ in these parts of $\pi$, and so defines a partition whose gauge is $|H|$.

It remains to show that different subgroups give different equivalence classes of partitions. So suppose that $H_1, H_2$ give equivalent partitions.

(i) Suppose $H_1 \leq H_2$ and that $H_1, H_2$ give equivalent partitions. Then $|H_1| = |H_2|$, and so $H_1 = H_2$.

(ii) In the general case, $H_1 \leq \langle H_1, H_2 \rangle$ and these subgroups give equivalent partitions; so $H_1 = \langle H_1, H_2 \rangle$. Similarly $H_2 = \langle H_1, H_2 \rangle$. So $H_1 = H_2$. $\dashv$

The injection cannot be a bijection because as we have noted above for as simple a case as a gauge-3 partition, the symmetric group quotient can have either order 1 or 3. This result picks out a sublattice of the lattice of equivalence classes of partitions. It may be interesting to study this in future work.

**§3. The centraliser of the gauge group.** Truss [6] showed that a bounded amorphous set of gauge $n$ is determined, up to a notion of equivalence which is somewhat difficult to explain, by two parameters: the *gauge group* $G_0$ (a transitive subgroup of the symmetric group $\mathrm{Sym}(n)$), and the *excess* (an integer from the set $\{0,\ldots,n-1\}$). In the this section, by examining Truss's classification more closely, we will identify the finite group $\mathrm{Sym}(U)/\mathrm{FSym}(U)$ with the centraliser of the gauge group.

There are two aspects of Truss's work. First, he shows how the gauge group and the excess are determined by the set $U$. (The excess is simply the number of parts of size different from $n$ in a standard form partition of gauge $n$. The construction of the gauge group is outlined below.) Second, he constructs a model of Fraenkel–Mostowski set theory in which the set of atoms is amorphous and has any prescribed gauge group and excess, and shows how a corresponding model of ZF can be obtained using the method of Jech and Sochor [2].

Note that, for any infinite set $U$, and any point $u \in U$, we have

$$\mathrm{Sym}(U)/\mathrm{FSym}(U) \cong \mathrm{Sym}(U\setminus\{u\})/\mathrm{FSym}(U\setminus\{u\}),$$

since $\mathrm{FSym}(U)$ is transitive and so meets every coset of $\mathrm{Sym}(U\setminus\{u\})$ in $\mathrm{Sym}(U)$. So it is enough to consider sets with excess equal to zero.

We now describe the gauge group. More details are given on [6, p. 199].

For every $\xi \in \pi$ let $\xi^*$ be the set of orderings of $\xi$ (so that $|\xi^*| = n!$ for all but finitely many members of $\pi$.) Now $\pi^* := \cup\{\xi^* : \xi \in \pi\}$ need not be amorphous but must have amorphous subsets because as $\pi$ is amorphous any $X \subseteq \pi^*$ has a unique $m \in \mathbb{Z}^+$ such that $\{\xi \in \pi : |X \cap \xi^*| = m\}$ is infinite. Choosing minimal $m$ makes $X$ amorphous; fix such an $X$. Let $g \in \mathrm{Sym}(n)$ act to fix all sequences in $\pi^*$ whose length is not $n$, and permute the entries appropriately for those of length $n$. For each $\xi \in \pi$ let $G_\xi$ be the set of members of $\mathrm{Sym}(n)$ preserving $X \cap \xi^*$ setwise. Since $\pi$ is amorphous there is a unique (up to isomorphism) permutation group occuring infinitely often as $G_\xi$; denote it $G_X$. If $X'$ is another amorphous subset of $\pi^*$, then $G_X$ and $G_{X'}$ are conjugate. Let $G_\pi$ be equal to $G_X$ for some amorphous subset $X$ of $\pi^*$. Then $G_X$ is uniquely determined by $\pi$ up to conjugacy in $\mathrm{Sym}(n)$ and so $G_\pi$ is well-defined. Moreover, the same conjugacy class of subgroups is determined by any other maximal partition $\pi'$; so the *gauge group* $G_\pi$ (up to conjugacy) is determined by $U$, so we can write it as $G(U)$. It is shown in [6, p. 199] that $G(U)$ acts transitively on $\{0, 1, \dots, n-1\}$.

THEOREM 3.1 (Main Theorem). *Let $U$ be a bounded amorphous set having gauge $n = n(U)$ and gauge group $G = G(U)$. Then*

$$\mathrm{Sym}(U)/\,\mathrm{FSym}(U) \cong C_{\mathrm{Sym}(n)}(G).$$

PROOF. As explained, the proof of the theorem relies on the construction of bounded amorphous sets by Truss. We outline the construction, referring to his paper [6] for an exposition of the method. It is also possible to prove the theorem using the description above of the gauge group; but we need the construction to show that every finite group can occur here.

We begin with a model $\mathcal{M}$ of Fraenkel–Mostowski set theory with choice (FMC); a modification of ZFC allowing a set $U$ of atoms. A *permutation model* $\mathcal{N}$ is constructed using a permutation group $G$ on $U$ and a filter $\mathcal{F}$ of subgroups of $G$. The model $\mathcal{N}$ consists of all 'hereditarily symmetric' sets; it is defined recursively by the rule

$$\mathcal{N} = \{x \in \mathcal{M} : x \subseteq \mathcal{N} \text{ and } G_x \in \mathcal{F}\},$$

where $G_x$ denotes the setwise stabiliser of $x$ in $G$. (The action of $G$ on $\mathcal{M}$ is also defined recursively.) We use $G_{(x)}$ to denote the pointwise stabiliser of $x$, that is, the intersection of the stabilisers of all members of $x$. Then it can be shown that, under suitable conditions, $\mathcal{N}$ is a model of FM.

Usually it is assumed that the filter $\mathcal{F}$ contains the stabilisers of the elements of $U$. This guarantees that the elements of $U$ (and hence $U$ itself) all belong to the model $\mathcal{N}$.

It can also be shown that, under suitable hypotheses, $\mathcal{N}$ can be 'transferred' to a model of ZF with similar properties. This is the theorem of Jech and Sochor [2]. In all the cases that we need, the verification of the hypotheses has been done already (see Truss [6]), so we have nothing to do in this connection.

The proof of our main theorem uses the following general result.

PROPOSITION 3.2. *Suppose that a permutation model $\mathcal{N}$ is defined by a set $U$ of atoms, a permutation group $G$ on $U$, and a filter $\mathcal{F}$ of subgroups of $G$ including the point stabilisers. Then $C_{\mathrm{Sym}(U)}(G) \in \mathcal{N}$.*

PROOF. Since $\mathcal{F}$ includes the point stabilisers, the points of $U$ belong to $\mathcal{N}$, and so (since $\mathcal{N}$ satisfies the axioms of FM) all ordered pairs $(u, v)$ of elements of $U$ lie in $\mathcal{N}$.

Take $h \in H = C_{\mathrm{Sym}(U)}(G)$. Then $h = \{(u, uh) : x \in U\}$. For any $g \in G$ and $u \in U$, we have

$$(u, uh)g = (ug, uhg) = (ug, ugh) = (v, vh) \in h,$$

where $v = ug$; so $G_h = G \in \mathcal{F}$. Hence $h \in \mathcal{N}$.

Finally, all elements of $H$ beling to $\mathcal{N}$, and $G_H \geq G_{(H)} = G$, so $H \in \mathcal{N}$, and the proof is done. ⊣

Truss [6] showed the following result. Here $G_0 \operatorname{Wr} \mathrm{Sym}(\omega)$ denotes the complete wreath product, that is the semidirect product of $B$ with $\mathrm{Sym}(\omega)$, where the base group $B$ is the cartesian product of $\omega$ copies of $G_0$.

PROPOSITION 3.3. *Let $G_0$ be a transitive subgroup of the symmetric group $\mathrm{Sym}(n)$, and let $U = n \times \omega$, $G = G_0 \operatorname{Wr} \mathrm{Sym}(\omega)$, and $\mathcal{F}$ the filter generated by the point stabilisers in $G$. Then in the corresponding permutation model, $U$ is a bounded amorphous set with gauge $n$, gauge group $G_0$, and excess $0$.*

We also need the following technical result.

PROPOSITION 3.4. *With the notation of the preceding proposition, $C_{\mathrm{Sym}(U)}(G) \cong C_{\mathrm{Sym}(n)}(G_0)$.*

PROOF. It is clear that, if $H_0 = C_{\mathrm{Sym}(n)}(G_0)$, and we extend the action of $H_0$ to $U = n \times \omega$ by letting it act on the first factor, then $H_0 \leq C_{\mathrm{Sym}(U)}(G)$.

Conversely, take any element $h$ of $H = C_{\mathrm{Sym}(U)}(G)$. We claim first that $h$ fixes every set $U_i = n \times \{i\}$ for $i \in \omega$. For there is a factor $G_i$ of the base group of $G = G_0 \operatorname{Wr} \mathrm{Sym}(\omega)$ which acts transitively on $U_i$ and fixes all points of $U_j$ for $j \neq i$; if $h$ maps $(x, i)$ to $(y, j)$ for some $j \neq i$, then the condition $gh = hg$ for $g \in G_i$ would imply that every point of $U_i$ would be mapped to $(y, j)$, a contradiction.

Now, using the fact that $h$ commutes with the top group $\mathrm{Sym}(\omega)$, we find that $h$ acts in the same way on each set $U_i$; so $h \in H_0$, and the proof is complete. ⊣

Combining these results gives the proof of the Theorem. ⊣

Using this, we can show that every finite group arises as the quotient $\operatorname{Sym}(U)/\operatorname{FSym}(U)$ for some bounded amorphous set $U$.

LEMMA 3.5. *Let $H$ be a semiregular subgroup of $S = \operatorname{Sym}(n)$. Then, excluding the case $n = 2, H = 1$, we have that $C_S(C_S(H)) = H$.*

PROOF. We begin with the simple observation that, if $G$ is transitive and $H$ commutes with $G$, then $H$-orbits are blocks of imprimitivity for $G$.

Now let $H$ be semiregular. Certainly $H$ is contained in $K = C_S(C_S(H))$. Moreover, $C_S(H)$ is transitive, so $K$ is semiregular. So we are done if we can show that $H$ and $K$ have the same orbits.

Now $C_S(H)$ induces the symmetric group on the set of $H$-orbits, and the set of $H$-orbits contained in a fixed $K$-orbit is a block of imprimitivity for this group. Since the symmetric group is primitive, if the conclusion does not hold, then $K$ must be transitive. But in this case, $C_S(H)$ is semiregular, and hence regular, which is not possible unless $H$ is transitive; and if $H$ is transitive then its orbit is the same as the orbit of $K$, so we are done. If $H$ is not transitive, let $\Delta_1, \ldots, \Delta_r$ be its orbits, then $C_S(H) \geq C_{\operatorname{Sym}(\Delta_1)}(H^{\Delta_1}) \times \ldots \times C_{\operatorname{Sym}(\Delta_r)}(H^{\Delta_r})$, where each of the $H^{\Delta_i}$ (for $1 \leq i \leq r$) are regular. This direct product is not semiregular unless $|\Delta_i| = 1$ and $H = 1$. So the theorem goes through unless $n = 2$, for when $H = 1$, $C_S(H) = \operatorname{Sym}(n)$ and so $C_S(C_S(H)) = 1$ if $n > 2$. ⊣

COROLLARY 3.6. *Given $n > 2$, $E < n$ and a semiregular subgroup $H \leq \operatorname{Sym}(n)$ then there exists a bounded amorphous set $U$ with gauge $n$ and excess $E$ and $\operatorname{Sym}(U)/\operatorname{FSym}(U) \cong H$.*

PROOF. Let $G = C_{\operatorname{Sym}(n)}(H)$. Then $G$ is transitive on $\{0, \ldots, n-1\}$. Applying the Truss construction, we obtain an amorphous set $U$ with $\operatorname{Sym}(U)/\operatorname{FSym}(U) \cong C_{\operatorname{Sym}(n)}(G) = H$. ⊣

Obviously, any finite group of order dividing $n$ can be embedded as a semiregular subgroup in $\operatorname{Sym}(n)$.

Remark. The results of this paper are taken from the second author's Ph.D. thesis [5].

REFERENCES

[1] P. J. CAMERON, ***Permutation Groups***, Cambridge University Press, Cambridge, 1999.

[2] T. Jech and A. Sochor, *On Θ-model of the set theory*, ***Bull. Acad. Pol. Sci., Ser. Sci. Math. Astron. Phys.***, vol. 14 (1966), pp. 297–303.

[3] P. Creed and J. K. Truss, *On o-amorphous sets*, ***Annals of Pure and Applied Logic***, vol. 101 (2000), no. 2–3, pp. 185–226.

[4] ———, *On quasi-amorphous sets*, ***Archive for Mathematical Logic***, vol. 40 (2001), pp. 581–596.

[5] S.Tarzi, *Group actions on amorphous sets and reducts of coloured random graphs*, ***Ph.D. thesis***, University of London, 2002.

[6] J. K. Truss, *The structure of amorphous sets*, ***Annals of Pure and Applied Logic***, vol. 73 (1995), no. 2, pp. 191–233.

SCHOOL OF MATHEMATICAL SCIENCES
QUEEN MARY, UNIVERSITY OF LONDON
MILE END ROAD
LONDON E1 4NS, UK
*E-mail*: P.J.Cameron@qmul.ac.uk