Review of *The Unfinished Game: Pascal, Fermat and the Seventeenth-Century Letter that Made the World Modern* by Keith Devlin

> For [Luck] your science finds no measuring-rods,

wrote Dante in the *Inferno* (in Dorothy L. Sayers' translation). For most of the history of humanity, few would have disagreed. Luck, or Chance, seems to be beyond the reach of our investigation, because it is the domain of the gods. (The best we could do was to study the outcomes of some random events such as livers of animals or flights of birds to try to divine the gods' intentions.)

Keith Devlin's thesis is that all this changed on Monday, August 24, 1654, when Blaise Pascal wrote a letter to Pierre de Fermat. This letter is the birth certificate of probability theory, the science of luck, which in Devlin's words allows us to "predict the future by calculating, often with extraordinary precision, the numerical *likelihood* of a particular event's occurring." Although I have no idea whether I will win on my next play at the roulette wheel, or when my death will come, the calculus of probability allows casinos and insurance companies to estimate their profits very accurately.

Pascal and Fermat, two of the giants of French mathematics at the time, were discussing a problem raised by Pascal's friend, the Chevalier de Méré. Suppose two people toss a fair coin, the best of seven to win. The game is interrupted when A has won two and B one. How should the prize be divided between them? (Try it for yourself; answer at the end.) Devlin has selected the most significant letter from their correspondence about this problem, and goes through it in detail to explain what they achieved.

The book puts the letter into context, tracing the history of probability theory from the first (flawed) discussion of the problem of the points by Luca Pacioli in 1494 to the stochastic differential equation devised by Fischer Black and Myron Scholes in 1970 for determining the price of stock options. He takes in the first statistical analysis of London mortality data by John Graunt in 1662 and the invention of expectation by Christiaan Huygens in 1669.

Pascal gives an algebraic solution to the problem, Fermat a combinatorial one. Although the invention of probability theory is usually credited to Pascal, it was Fermat who was the stronger of the two; we see Pascal groping his way to an understanding of Fermat's (superior) method, and Fermat's dismissive reply.

With hindsight it is clear that Fermat's approach leads to describing all possible outcomes of an experiment, defining events as subsets of the set of outcomes, and assigning probabilities to them (the modern measure-theoretic foundations of the subject). Pascal's approach is more a computational tool, and leads on to random walks and the Black–Scholes equation.

I feel that Devlin overstates his case that probability makes the future predictable. If we could predict the future, then we could become rich. Devlin rightly warns that you can't just enter the derivatives market armed with Black–Scholes software and become rich. It would be instructive to ask why this is. One reason commonly advanced is that, when almost all brokers are using this software, the assumption of independence in the model is unlikely to be true.

The seventeenth century was a time of huge scientific advance. Even in mathematics, the invention of probability was overshadowed by that of calculus. It is interesting to see these two huge areas come together in the Black–Scholes equation. This shows that it is very difficult to untangle the contribution of a single letter to the creation of the modern world.

In the problem of the points, the stake should be divided in the ratio 11 to 5.

Peter J. Cameron