Wiggles and finitely discontinuous k-to-1 functions between graphs

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Abstract. The graphs we shall consider are topological graphs - that is they lie in $\mathbb{R}_3$ and each edge is homeomorphic to $[0, 1]$. If a graph is simple, that is it has no loops or multiple edges, then each edge may be taken to be a straight line joining the two vertices at the ends of the edge.

A function is k-to-1 if each point in the codomain has precisely k pre-images in the domain. Given two graphs G and H, and an integer $k \geq 1$, Jo Heath proved the surprising result that there exists a finitely discontinuous k-to-1 function f from G onto H if and only if

$$\begin{array}{rrcll} & |E(G)| - |V(G)| & \leq & k\,(|E(H)| - |V(H)|) & \text{if} \quad k \geq 3\ , \\ \text{and} & |E(G)| - |V(G)| & = & 2\,(|E(H)| - |V(H)|) & \text{if} \quad k = 2\ . \end{array}$$

Such functions often involve a limit construction, which we call a *wiggle*. In this talk, I shall discuss a simple formula (related to Jo Heath's result) which counts the number of wiggles.

I shall also discuss the special case when the finitely discontinuous function f can actually be chosen to be continuous. Much of the talk will be joint work from the past with Jo Heath, or current work with my student, John Gauci.