CAUSAL SET GLOSSARY 2001 Nov 20

- order = poset = ordered set = partially ordered set

  An order is a set of elements carrying a notion of “ancestry” or “precedence”. Perhaps the simplest way to express this concept axiomatically is to define an order as a transitive, irreflexive relation $<$. Many other, equivalent definitions are possible.

  It seems convenient to admit the empty set as a poset.

- locally finite

  An order is locally finite iff all its order-intervals are finite. (‘interval’ is defined below)

- past-finite

  An order is past-finite iff all its down-sets are finite. (‘down-set’ is defined below)

- causet = causal set = locally finite order

- preorder = preposet = acyclic relation = acyclic digraph

  In other words, a preorder is a relation whose transitive closure is an order.

- pseudo-order = transitive relation (possibly with cycles)

- link = covering relation

  An irreducible relation of an order, that is, one not implied by the other relations via transitivity. Of course we exclude pairs (x x) from being links, in the case where such pairs are admitted into ¡ at all.

- child/parent

  If $x < y$ is a link we can say that x is a parent of y and y a child of x.

  Many people also say ”y covers x” for this relationship.

- ancestor/descendant

  If $x < y$ then x is an ancestor of y and y is a descendant of x.

- past/future

  $(\text{past}\, x) = \{y \mid y < x\}$, $(\text{future} x) = \{y \mid x < y\}$.
  Can also write Px and Fx for short.

- down-set = past-set = order-ideal = ancestral set

  A subset of an order that contains all the ancestors of its members

- partial stem

  A past set of finite cardinality
  Also called just 'stem'

- full stem

  A partial stem whose complement is the (exclusive) future of its top layer

- order-interval (or interval)

  This can be either exclusive or inclusive of it endpoints, in either case we regard the endpoints as part of the definition of the interval.

- isotone map

  A function f from one order to another such that $x < y \Longrightarrow f(x) < f(y)$.
  (an "order-morphism")

- order-isomorphic

  Isomorphic as posets.
  Do we want a special word for this, eg tonomorphic, toneomorphic?

- antichain

  A trivial order in which no element is related to any other

- chain = linear order

  In particular, any linearly ordered subset of an order is a chain.
  n-chain = chain of *n* elements.

- path = saturated chain

  I.e. a chain all of whose links are also links of the enveloping poset
  (saturated means it might be "enlarged" but it can't be "filled in")

- slice = maximal antichain

  where maximal means it can't be enlarged and remain an antichain (could also say "saturated antichain" for this)
  equivalently, every *x* in the causet is either in the slice or comparable to one of its elements.
  equivalently, its inclusive past is a full stem

- origin = minimum element

  A single element which is the ancestor of all others

- post

  An element such that every other element is either its ancestor or its descendant, i.e. a one-element slice.

- partial post

  An element *x* of which no descendant has an ancestor spacelike to *x*, i.e. *x* is a post for PFx or $PFx \subseteq (PxcupFxcup\{x\})$.
  The idea is x is the progenitor of a "child universe"

- related = comparable

Two elements x and y are 'related' (or 'comparable') if $x < y$ or $y < x$.

- spacelike = incomparable

  Two elements x and y are spacelike iff they are unrelated (i.e. neither $x < y$ nor $y < x$)

  The notation for this is the ”natural” symbol $\natural$

- level

  In a past-finite causet the level of an element x is the number of links in the longest chain $a < b < \ldots < c < x$. Thus, level 0 comprises the minimal elements, level 1 is level 0 of the remainder, etc.

- natural labeling

  A natural labeling of a past-finite order is an assignment to its elements of labels $0, 1, 2, \ldots$ such that $x < y \Longrightarrow \text{label}(x) < \text{label}(y)$. Thus it is essentially the same thing as a locally finite linear extension.

- sup = least upper bound
- inf = greatest lower bound
- transtively reduced

  The “transitive reduction” of an order is its Hasse digraph, an acyclic relation containing only links.