Problems from the Eighteenth British Combinatorial Conference

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Abstract

Most of these problems were presented at the problem session of the 18th British Combinatorial Conference at the University of Sussex, 2–6 July 2001. I have added some problems given to me after the session.

Since two of the problems concern circular chromatic number, the entire set has a circular structure; the starting point is fixed by the sixth problem.

Problem 1 (BCC18.1) Freese–Nation numbers of posets. Proposed by D. H. Fremlin and D. B. Penman.

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Let $(P,\preceq)$ be a poset. A function $f : P \mapsto \mathcal{P}P$ (where $\mathcal{P}P$ is the power set of P) is a *Freese–Nation function* if, whenever $p \preceq q$, we have

$$f(p) \cap f(q) \cap [p,q] \neq \emptyset.$$

The *Freese–Nation number* $\mathrm{FN}(P)$ is the smallest r for which there is a Freese–Nation function f with $|f(p)| < r$ for all $p \in P$. Observe that $p \in f(p)$ for all $p \in P$.

For example,

1. if P is an antichain, then $\mathrm{FN}(P) = 2$;
2. if P is an n-element chain, then $\mathrm{FN}(P) = 2 + \lfloor \log_2 n \rfloor$;
3. if $P = A \cup B$ with $|A| = 2r-5$, $|B| = 2r-6$, and $a \preceq b$ for all $a \in A$, $b \in B$, then $\mathrm{FN}(P) = r$;
4. If P is selected from the uniform distribution on n-element posets, then $\mathrm{FN}(P) = (n/8)(1 + o(1)$ with high probability.

Problem: Find

$$\lim_{m \to \infty} (\mathrm{FN}(\mathcal{P}m))^{1/m},$$

where m denotes an m-element set. (It is known that the limit exists and lies in the interval $[2/\sqrt{3}, \sqrt[3]{3}] \approx [1.1547, 1.4422]$.)

Problem 2 (BCC18.2) Matching roots of vertex-transitive graphs. Proposed by Bojan Mohar.

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Let $p(G,k)$ be the number of matchings of the graph G with k edges. Then the *matching polynomial* of G is

$$\mu(G,x) = \sum_{k=0}^{\lfloor n/2 \rfloor} (-1)^k p(G,k) x^{n-2k}.$$

It is known that $\mu(G,k)$ has only real roots.

Conjecture: For every integer r there exists a connected vertex-transitive graph whose matching polynomial has a root of multiplicity at least r.

Even examples of vertex-transitive graphs with at least one non-simple root would be of great interest, since such graphs cannot contain a Hamiltonian path (see [12]).

Editor's note: This was the proposer's "Problem of the month" for July 2001: see `http://www.fmf.uni-lj.si/~mohar/Problems.html`

Problem 3 (BCC18.3) Strongly distance-regular graphs. Proposed by M. A. Fiol.

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For the definition of a distance-regular graph and related concepts, we refer to Brouwer *et al.* [4].

A graph Γ with diameter d is called *strongly distance-regular* if Γ is distance-regular and the distance-d graph Γ_d (in which vertices are adjacent if they have distance d in Γ) is strongly regular. Examples include

1. any strongly regular graph;
2. any distance-regular graph with $d = 3$ and third-largest eigenvalue -1;
3. any antipodal distance-regular graph.

Problem: Prove or disprove that these examples exhaust all possibilities.

Problem 4 (BCC18.4) Some configurations in polar spaces. Proposed by Harm Pralle.

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For which polar spaces Π of rank 3, other than the Klein quadric, does there exist a set H of planes such that

(i) there exists a unique plane $\delta \in H$ such that any plane of Π intersecting δ in a line belongs to H, and

(ii) every line of Π not contained in δ is covered uniquely by a plane of H?

The only known example for H lives in the symplectic variety $S_5(\mathbb{R})$ in $\mathrm{PG}(5,\mathbb{R})$; it is a hyperplane of the dual of $S_5(\mathbb{R})$ arising from an embedding in $\mathrm{PG}(13,\mathbb{R})$. (All examples in the Klein quadric are obtained by taking δ to be a plane and including also all the planes of the opposite ruling.)

Problem 5 (BCC18.5) Projective space analogues of Steiner systems.
Proposed by "Folklore" (possibly Ph. Delsarte).

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Does there exist a collection S of planes in the projective space $\mathrm{PG}(n,q)$, where $n > 2$, such that any line lies in a unique member of S? (This would be the analogue for projective spaces of a Steiner triple system.) No examples are known.

One can easily define analogues of arbitrary t-designs in projective spaces (probably Delsarte [6] was the first to do so), but very few examples are known. However, infinite examples exist in great profusion!

Problem 6 (BCC18.6) "Problem 6". Proposed by Harald Gropp.

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Is there a bipartite 6-regular graph with 66 vertices having girth 6?

Equivalently, is there a 33_6 configuration? (This is a configuration with 33 points and 33 lines, each point on 6 lines and each line containing 6 points, such that two points lie on at most one line.)

Problem 7 (BCC18.7) Multiplication group of a Latin square. Proposed by Aleš Drápal.

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Consider a Latin square L of order n whose first row and column are normalised to have the entries $1,\ldots,n$ in order. Each row and column of L is a permutation of $\{1,\ldots,n\}$; the group generated by these permutations is the *multiplication group* of L, denoted by $M(L)$.

Given $k \geq 3$ and a prime power q, does there exist a Latin square L such that

$$\mathrm{PSL}(k,q) \leq M(L) \leq \mathrm{P\Gamma L}(k,q)?$$

The proposer has shown recently [7, 8] that, if $k=2$, there is only one such square L, with $M(L) = \mathrm{P\Gamma L}(k,q) = S_5$.

For the next two problems, we introduce a Markov chain method for choosing Latin squares uniformly at random, due to Jacobson and Matthews [13].

We represent a Latin square of order n by a function $f : N^3 \to \{0,1\}$ (where $N = \{1,\ldots,n\}$) satisfying

$$\sum_{x \in N} f(x,y,z) = 1$$

for given $y,z \in N$, and two similar equations for the other coordinates. We allow also *improper Latin squares*, which are functions satisfying these constraints but which take the value -1 exactly once. Now to take one step in the Markov chain starting at a function f, do the following:

(a) If f is proper, choose any (x,y,z) with $f(x,y,z) = 0$; if f is improper, use the unique triple with $f(x,y,z) = -1$.

(b) Let $x',y',z' \in N$ satisfy

$$f(x',y,z) = f(x,y',z) = f(x,y,z') = 1.$$

(If f is proper, these points are unique; if f is improper, there are two choices for each of them.)

(c) Now increase the value of f by one on the triples (x,y,z), (x,y',z'), (x',y,z') and (x',y',z), and decrease it by one on the triples (x',y,z), (x,y',z), (x,y,z') and (x',y',z'). We obtain another proper or improper Latin square, according as $f(x',y',z') = 1$ or 0.

Jacobson and Matthews show that the limiting distribution gives the same probability to each Latin square.

Problem 8 (BCC18.8) Choosing Latin squares uniformly at random.
Proposed by M. T. Jacobson and P. Matthews; J. Møller; J. Besag.

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Problem: How fast does the Jacobson–Matthews Markov chain converge to the uniform distribution?

Problem 9 (BCC18.9) A Markov chain for Steiner triple systems. Proposed by Peter J. Cameron.

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A slight modification of the method of Jacobson and Matthews should work for Steiner triple systems. We simply replace "ordered triples" by "unordered triples of distinct elements" in the definition; then a STS is a function from unordered triples to $\{0,1\}$ which satisfies

$$\sum_{z \neq x,y} f(\{x,y,z\}) = 1$$

for all distinct points x, y, and an improper STS is allowed to take the value -1 exactly once. Now the moves are defined as before. However, before we know that the limiting distribution is uniform, we have to solve the following

Problem: Is the chain connected? That is, is it possible to get from any STS to any other by a sequence of moves?

Problem 10 (BCC18.10) Perfect Steiner triple systems. Proposed by M. J. Grannell and T. S. Griggs.

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Let $S = (V,B)$ be a Steiner triple system of order v, and let a and b be any two points, and c the third point of the block containing them. Define a graph G_{ab} as follows: the vertex set is $V \setminus \{a,b,c\}$, and two vertices x and y are adjacent if and only if either $\{a,x,y\} \in B$ or $\{b,x,y\} \in B$. Clearly G_{ab} is a union of disjoint even cycles. If G_{ab} is a single cycle for *all* choices of $a,b \in V$, then S is said to be *perfect*.

Perfect STS of orders 7, 9, 25 and 33 have been known for some time. More recently Grannell, Griggs and Murphy [11] added nine new values to the list of orders:

$$79, 139, 367, 811, 1531, 25771, 50923, 61339, 69991.$$

These are all primes of the form $12s+7$.

Problem: What number-theroretic property distinguishes these nine primes from the other primes of this form less than 100000 (where the search terminated)?

The next two problems refer to circular chromatic number, which is defined as follows. For a hypergraph H, and positive integers p,q with $2q \leq p$, we define a (p,q)-*colouring* to be a function $c : V(H) \to \{0,1,\ldots,p-1\}$ such that each edge e of H contains two vertices a and b with $q \leq |c(x) - c(y)| \leq p - q$. The *circular chromatic number* of H, written $\chi_c(H)$, is the infimum of the set of values p/q for which there exists a (p,q)-colouring of H. (We can replace "inf" by "min" here.) Alternatively, it is the smallest circumference of a circle S such that the vertices of the graphs can be mapped to points of S such that adjacent points are at distance at least 1. The definition of circular chromatic number of a graph is just the specialisation of this definition.

Since every p-colouring is a $(p,1)$-colouring, we have $\chi_c(H) \leq \chi(H)$, where $\chi(H)$ is the chromatic number of H.

See Zhu [20] for a survey, and also the paper by Mohar [15] presented at the meeting.

Problem 11 (BCC18.11) Circular chromatic number of Steiner triple systems. Proposed by Changiz Eslahchi, Arash Rafiey.

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Conjecture: For every Steiner triple system S of order at least 13, we have $\chi_c(S) = \chi(S)$.

Editor's note: The conjecture is false for order 7. I am grateful to Fred Holroyd for pointing out to me that the usual cyclic representation of the STS of order 7 shows that $\chi_c(S) \leq 7/3$, while of course $\chi(S) = 3$.

Problem 12 (BCC18.12) Bounding the circular chromatic number of a graph. Proposed by Bojan Mohar.

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Let $P_G(x)$ be the chromatic polynomial of the graph G and let k be the chromatic number of G. Let $c_0 \leq k$ be the largest real number such that $P_G(c_0) = k!$
.

Problem: Is it true that $\chi_c(G) \leq c_0$, where $\chi_c(G)$ is the circular chromatic number of G?

Problem 13 (BCC18.13) Two list colouring conjectures. Proposed by S. Akbari, V. S. Mirrokni, B. S. Sadjad.

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1. *A list edge-colouring conjecture.* Let G be a graph with m edges and maximum degree $\Delta \geq 2$. Suppose that $L = \{L_1, \ldots, L_m\}$ is an assignment of lists of colours to the edges of G such that $|L_i| = \Delta$ for $i = 1, \ldots, m$. Show that G is *not* uniquely L-colourable.

This is known to be true if G is not regular, or if G is regular and bipartite (see [3]).

2. *A list vertex-colouring conjecture.* Suppose that G is a graph and $f : V(G) \longrightarrow \mathbb{N}$ is a function, where $\mathbb{N}$ is the set of natural numbers. Let L be a list assignment to the vertices of G, such that $|L_v| = f(v)$ for any $v \in V(G)$, and assume that G is uniquely L-colourable. Suppose that G is a maximal uniquely f-colorable graph (that is, for any list assignment L' of G, if $f(v) \leq |L'_v|$ for all $v \in V(G)$ and there exists a vertex v_0 such that $f(v_0) < |L'_{v_0}|$, then G is not uniquely L'-colorable). Then G is f-choosable.

Problem 14 (BCC18.14) Colouring and degeneracy of random graphs.
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Here $\mathcal{G}_{n,p}$ denotes the random graph model in which edges are selected from the n-vertex set independently with probability p (see Molloy's paper [16] presented at the conference). A graph is k-degenerate if every induced subgraph has a vertex of degree smaller than k. Clearly a k-degenerate graph is k-colourable. A k-*core* of a graph is an induced subgraph with minimum degree at least k.

Let $p = p(n,k)$ be the smallest probability such that almost no graphs in $\mathcal{G}_{n,p}$ are $(k \log k)$-degenerate.

Conjecture: Almost all graphs in $\mathcal{G}_{n,p}$ have chromatic number at least k. (In other words, the threshold for a $(k \log k)$-core is at least that for k-colourability.)

Problem 15 (BCC18.15) Odd holes in planar graphs. Proposed by Colin McDiarmid.

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An *odd hole* in a graph is an induced subgraph which is an odd circuit of length at least 5.

Does every planar graph have 3-colouring (not necessarily proper) of the vertices such that every odd hole receives all three colours?

This question is related to measuring how imperfect a planar graph can be.

Problem 16 (BCC18.16) Chord diagrams and Vassiliev invariants. Proposed by Leonid Plachta.

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The following combinatorial problem arises in the study of Vassiliev knot invariants. To formulate it let us first recall that each n-singular knot (C^1-immersion of S^1 into $\mathbb{R}^3$) with exactly n double transverse points (called singularities) can be represented (though not uniquely) by its *chord diagram* (for short, CD), in which the preimages of each singular point in S^1 are the endpoints of a chord in the CD.

Let $\mathcal{K}$ denote the set of knots in $\mathbb{R}^3$. Any isotopy invariant of knots $v\colon \mathcal{K} \to \mathbb{Q}$ can be extended in a natural way to the set $\mathcal{L}$ of singular knots with a finite number of singularities (see, for example, [2]). An isotopy invariant $v\colon \mathcal{L} \to \mathbb{Q}$ is called a *Vassiliev invariant* of order n if v vanishes on any $(n+1)$-singular knot and n is the smallest number with this property. It turns out (see [2]) that any Vassiliev invariant v of order n has equal values on all singular knots having the same CDs with n chords.

Let D_n denote the set of chord diagrams with n chords, the CDs being considered up to the obvious equivalence relation, and let $\mathrm{span}(D_n)$ be the vector space over $\mathbb{Q}$ generated by D_n. It follows any $\mathbb{Q}$-valued Vassiliev invariant v of order

n determines a function $w(v)\colon D_n \to \mathbb{Q}$ satisfying the axioms 1T (framing independence) and 4T (the four term relation) described, for example, in [2]. Such a function is called a *weight system* of degree n. In other words, a weight system of degree n is an element of the dual space of the vector space

$$\mathcal{A}_n = \mathrm{span}(D_n)/\mathrm{span}(\{4\mathrm{T} \text{ and } 1\mathrm{T relations}\}).$$

For any $D \in D_n$, let $G(D)$ denote the *intersection graph* (or *interplay graph*, in the terminology of [1]) of D. Note that not every abstract intersection graph with n vertices is realizable as an intersection graph of some chord diagram of order n. Rosenstiehl's theorem characrerizes the class of all realizable abstract intersection graphs (see [1]).

The Intersection Graph Conjecture, formulated by Chmutov *et al.* [5], asserts that a weight system $w\colon D_n \to \mathbb{Q}$ has equal values on any two chord diagrams with the same intersection graphs, so its values on CDs are determined uniquely by their intersection graphs. They proved the conjecture in the case when the intersection graphs of chord diagrams are trees. It follows that the conjecture is true if the intersection graphs are forests. Recently B. Mellor [14] showed that the conjecture is true for chord diagrams whose intersection graphs have exactly one loop.

T. Q. T. Le showed however that, in general, the conjecture is false, since it implies that Vassiliev knot invariants cannot detect mutation, contradicting the Morton/Cromwell examples. More precisely, Morton and Cromwell [17] showed that there exists a framed Vassiliev invariant v of degree 11 with values in $\mathbb{Z}[u]$ which takes different values on Kinoshita-Teresaka/Conway mutants. This implies that there exists a (framing independent) $\mathbb{Q}$-valued Vassiliev invariant of order 11 distinguishing both the mutants (see [19]). This example yields two singular knots representing by CDs D_1 and D_2 of order 11, with the same intersection graphs $G(D_1)$ and $G(D_2)$, and such that $[D_1] \neq [D_2]$ in $\mathcal{A}_{11}$.

Problem: Describe the class of all (realizable) intersection graphs for which the Intersection Graph Conjecture is true.

Problem 17 (BCC18.17) Fragmentability of graphs of bounded degree.
Proposed by Keith Edwards, Graham Farr.

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Let C be a positive integer and α a real number in $(0,1)$. A graph G on n vertices is (C,α)-*fragmentable* if there exists a set X of at most αn vertices such that each component of $G-X$ has at most C vertices.

Problem: Does there exist $\alpha < 1$ and a sequence $C_1, C_2, \ldots$ of constants such that every graph G of maximum degree Δ is (C_Δ, α)-fragmentable?

It is known that such an α must be at least $1/2$: see [9].

Problem 18 (BCC18.18) Monotone paths in edge-ordered graphs. Proposed by Yehuda Roditty.

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An *edge-ordered graph* is an ordered pair (G, f), where $G = G(V,E)$ is a finite undirected simple graph and f is a bijection from $E(G)$ to $\{1,2,\ldots,|E(G)|\}$, called an *edge-ordering* of G. A *monotone path of length* k in (G, f) is a simple path $P_{k+1} : v_1, v_2 \ldots, v_{k+1}$ in G such that the values $f((v_i, v_{i+1}))$, for $i = 1,2,\ldots,k-1$, are strictly monotonic (either increasing or decreasing). All definitions and updated results can be found in [18].

Given a graph G, denote by $\alpha(G)$ the minimum (over all edge orderings of G) of the maximum length of a monotone path.

Problems:

1. Prove that $\alpha(K_n) = (\frac{1}{2} + o(1))n$. (The right-hand side is known to be an upper bound for $\alpha(K_n)$.)

2. Determine $\alpha(G)$ for G a planar graph. (It is known that $5 \leq \alpha(G) \leq 9$, and if G is bipartite then $4 \leq \alpha(G) \leq 6$).

Problem 19 (BCC18.19) Decomposing complete multipartite graphs.
Proposed by Keith Edwards.

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A graph H *decomposes* a graph G if there is a set S of subgraphs of G, each isomorphic to H, such that each edge of G is contained in exactly one of the graphs in S.

Problem: Is it true that, for any λ-partite graph H, there is an integer n such that H decomposes the complete λ-partite graph with all parts of size n?

The answer is "yes" for $\lambda = 2$ and $\lambda = 3$.

Problem 20 (BCC18.20) Graphs isomorphic to their neighbourhoods and non-neighbourhoods. Proposed by Anthony Bonato.

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Let $N(x)$ and $N^c(x)$ denote the sets of neighbours and non-neighbours of the vertex x of a graph G, respectively. We say that G has *property* (N) if, for every vertex x, the subgraph induced by $N(x)$ is isomorphic to G; property (N^c) is defined similarly.

Problem Which countable simple graphs have *both* property (N) and property (N^c)?

The only known example of such a graph is the countable *random graph*, or *Rado's graph*, the unique countable existentially closed graph. However, there are $2^{\aleph_0}$ non-isomorphic graphs having one of these properties.

References

[1] M. Aigner, The Penrose polynomial of graphs and matroids, pp. 11–46 in *Surveys in Combinatorics, 2001* (ed. J. W. P. Hirschfeld), London Math. Soc. Lecture Notes **288**, Cambridge University Press, Cambridge, 2001.

[2] D. Bar-Natan, On the Vassiliev invariants, *Topology* **34** (1995), 423–472.

[3] O. V. Borodin, A. V. Kostochka and D. R. Woodall, List edge and list total colourings of multigraphs, *J. Combinatorial Theory* (B) **71** (1997), 184–204.

[4] A. E. Brouwer, A. M. Cohen and A. Neumaier, *Distance-regular Graphs*, Springer, Berlin, 1989.

[5] S. V. Chmutov, S. V. Duzhin and S. K. Lando, Vassiliev knot invariants II: Intersection Graph Conjecture for trees, *Adv. Soviet Math.* **21**, Singularities and Curves, (ed. V. I. Arnold) (1994), pp. 117–126.

[6] Ph. Delsarte, The association schemes of coding theory, *Philips Res. Repts. Suppl.* **10** (1973).

[7] A. Drápal, Multiplication groups of finite loops that fix at most two points, *J. Algebra* **235** (2001), 154–175.

[8] A. Drápal, Multiplication groups of loops and projective semilinear transformations in dimension two, submitted.

[9] K. Edwards and G. Farr, Fragmentability of graphs, *J. Combinatorial Theory* (B) **82** (2001), 30–37.

[10] C. D. Godsil and I. Gutman, On the theory of the matching polynomial, *J. Graph Theory* **5** (1981), 137–144.

[11] M. J. Grannell, T. S. Griggs and J. P. Murphy, Some new perfect Steiner triple systems, *J. Combinatorial Design* **7** (1999), 327–330.

[12] O. J. Heilman and E. H. Lieb, Theory of monomer-dimer systems, *Commun. Math. Phys.* **25** (1972), 190–232.

[13] M. T. Jacobson and P. Matthews, Generating uniformly distributed random Latin squares, *J. Combinatorial Design* **4** (1996), 405–437.

[14] B. Mellor, The Intersection Graph Conjecture for loop diagrams, *J. Knot Theory Ramif.* **9** (2000), N2.

[15] B. Mohar, Graph minors and graphs on surfaces, pp. 145–163 in *Surveys in Combinatorics, 2001* (ed. J. W. P. Hirschfeld), London Math. Soc. Lecture Notes **288**, Cambridge University Press, Cambridge, 2001.

[16] M. Molloy, Thresholds for colourability and satisfiability in random graphs and boolean formulae, pp. 165–197 in *Surveys in Combinatorics, 2001* (ed. J. W. P. Hirschfeld), London Math. Soc. Lecture Notes **288**, Cambridge University Press, Cambridge, 2001.

[17] H. R. Morton and P. C. Cromwell, Distinguishing mutants by knot polynomials, *J. Knot Theory Ramif.* **5** (1996), 225–238.

[18] Y. Roditty, B. Shoham and R. Yuster, Monotone paths in edge-ordered sparse graphs, *Discrete Math.* **226** (2001), 411–417.

[19] A. Stoimenov, Gauss sum invariants, Vassiliev invariants and braid sequences, *J. Knot Theory Ramif.*, to appear.

[20] X. Zhu, Circular chromatic number: a survey, *Discrete Math.* **229** (2001), 371–410.