Reading Old Codger's Combinatorics Colloquium

5th November 2008 at Reading University

The talks will be in the Right Hand Physics Lecture Theatre. Coffee and tea available in Room 112 of the Mathematics Department. Lunch may be obtained in the Cedar Room.

Programme

10.30 – 11.00	*Coffee*
11.00 – 11.10	Welcoming remarks by Professor S.N. Chandler-Wilde, Head of Mathematics at Reading
11.10 – 11.50	D.R. Woodall *Two conjectures on graph colourings*
12.00 – 12.40	D.A. Preece *Daisy chains with three generators*
12.40 – 14.00	*Lunch*
14.00 – 14.40	D.J.A. Welsh *Some open questions about graph polynomials*
14.50 – 15.30	M. Grannell *Biembeddings of Latin squares*
15.30 – 16.10	*Tea*
16.10 – 16.50	N.L. Biggs *A matrix method for flow polynomials*

The Old Codger's Combinatorics Colloquium is intended to provide a forum for the awkward period when compulsory retirement forces active combinatorial mathematicians out of having a proper role in the university system before they feel inclined to cease mathematical research.

All are welcome to attend. We thank the Mathematics Department at Reading for allowing this meeting to be an official departmental activity, and the British Combinatorics Committee for its financial support.

It is particularly noteworthy that three of the five speakers, namely Norman Biggs, Douglas Woodall and Dominic Welsh played a crucial role in shaping combinatorial activity in Britain for the past 40 years. They held the first conference, and they took the lead in forming the British Combinatorics Committee.

Abstracts

Two conjectures on graph colourings

Douglas Woodall (Nottingham)

In this talk I will discuss what is known about the following two conjectures.

(1) The *adjacent strong edge chromatic number* $\chi_{as}(G)$ of a graph G, a.k.a. its *neighbour-distinguishing index* ndi(G), is the smallest number of colours in a proper edge colouring of G such that no two adjacent vertices have identical sets of colours on their edges. The *total chromatic number* $\chi''(G)$ is the smallest number of colours with which one can colour the vertices and edges of G so that no two adjacent or incident elements get the same colour. It is conjectured that if G is a regular graph with no K_2 or C_5 component, then these two numbers are equal.

(2) It is conjectured that if G is a k-colourable graph with at least k vertices, and G is not a circuit with length congruent to 1 mod k, then G has a (vertex) k-colouring c and a spanning tree T such that $c|_T$ is *variegated* and *panconnected*, meaning that if c is regarded as a colouring of T then the colours are well mixed up and close together (in T), in senses to be made precise.

Daisy Chains with Three Generators

Donald Preece (Queen Mary)

For many positive odd integers n, whether prime power or composite, the set $\mathbb{U}_n$ of units of $\mathbb{Z}_n$ contains members u, v and w, say with respective orders ψ, ω and π, such that we can write $\mathbb{U}_n = \langle u\rangle \times \langle v\rangle \times \langle w\rangle$. Each element of $\mathbb{U}_n$ can then be written in the form $u^i v^j w^k$ where $0 \le i \le \psi - 1$, $0 \le j \le w - 1$ and $0 \le k \le \pi - 1$. We can then often use the structure of $\langle u\rangle \times \langle v\rangle \times \langle w\rangle$ to arrange the $\psi\omega\pi$ elements of $\mathbb{U}_n$ in a *daisy chain*, *i.e.* in a circular arrangement such that, as we proceed round the chain in either direction, the set of differences between each member and the preceding one is itself the set $\mathbb{U}_n$. We describe such daisy chains as *daisy chains with three generators*. Each such daisy chain consists of a succession of *super-segments* of length $\omega\pi$, each made of *segments* of length π. Within each segment, each successive element is obtained from the preceding one by multiplication by w; within each super-segment, each successive segment is obtained from the preceding one by multiplication by v; each successive super-segment is obtained from the preceding one by multiplication by u. Some of these arrangements can be obtained from general constructions, but others are obtained *ad hoc*. In many examples of these arrangements, one of the generators has order 2; if n is prime, that generator must then be −1 (mod n), but if n is composite, another square root of n may be used.

Some Open Questions about Graph Polynomials

Dominic Welsh (Oxford)

I shall discuss some less well known open problems about the chromatic and related polynomials of a graph.

Biembeddings of Latin squares

Mike Grannell (The Open University)

A Latin square may be regarded as a set of (row, column, entry) triples, and each triple may be taken to represent the vertices of a triangle. Given two Latin squares of the same order, having the same row labels, the same column labels, and the same set of entries, we can sew together the triangles along common edges. Sometimes an oriented surface (a sphere with an appropriate number of handles) results. We then say that the two squares are biembedded in the surface. More generally, we say that two Latin squares of the same order are biembeddable in a surface if representatives from their Main Classes can be biembedded in that surface.

In this talk I will describe results that show why some pairs of Latin squares of the same order are not biembeddable, and results showing that some pairs are biembeddable. However, a lot of open questions remain, some of which will be discussed.

The talk is based on joint work, principally with Diane Donovan, Terry Griggs, Martin Knor and James Lefevre.

A Matrix Method for Flow Polynomials

Norman Biggs (London School of Economics)

A method for calculating flow polynomials based on a transfer matrix is described. It is analogous to the method used for chromatic polynomials, although there is as yet no parallel development of the theory. The new method is applied to a family of bracelets, and the limit curves for the flow roots are obtained. There is an unexplained similarity between these calculations and the corresponding ones for the chromatic polynomials of the same family.

A provisional version of this talk is available in the CDAM Research Reports Series LSE-CDAM 2008-08