Queen Mary
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C50 Enumerative & Asymptotic Combinatorics

Prize question Spring 2003

A £10 prize is offered for the first correct solution.

1 The following problem arises in the theory of clinical trials. A new drug is to be tested. Of $2n$ subjects in the trial, n will receive the new drug and n will get a placebo. To avoid bias, it is important that the doctor recruiting patients to the trial cannot know, and cannot reliably guess, which treatment the next patient patient will receive. The patients enter the trial one at a time, and are numbered from 1 to $2n$.

If the treatments were allocated randomly with probability $1/2$, the doctor's guesses could be no better than random (so that the expected values for the numbers of correct and incorrect guesses are both n); but then the numbers of patients receiving drug and placebo would be unlikely to be equal. Given that they must balance, the doctor can certainly guess at least the last patient's treatment correctly.

If we allocated the drug and the placebo randomly to patients $2i-1$ and $2i$ for $i = 1, \ldots, n$, then the doctor can correctly guess the treatment for each even-numbered patient.

Suppose that instead we choose a random set of n patients to allocate the drug to, and the remaining n get the placebo; each of the $\binom{2n}{n}$ sets is equally likely. Suppose also that the doctor guesses according to the following rule. If the number of patients so far having the drug and the placebo are equal, he guesses at random about the next treatment. If the drug has occurred more often than the placebo, he guesses that the next treatment is the placebo, and *vice versa* if the placebo has occurred more often than the drug.

Find a formula, and an asymptotic estimate, for the expected value of the difference between the number of correct guesses and the number of incorrect guesses that the doctor makes.

Hint: First find the expected number of times during the trial when the numbers of patients allocated the drug and the placebo are equal.