MTH6140 Linear Algebra II

Assignment 1 For handing in on Monday 11th October 2010

Question 1 only *will be marked for credit. Write your name and student number at the top of your solution before handing it in. Post the solution in the* ***YELLOW*** *post-box in the* ***BASEMENT*** *of the Maths building before 15:00 on Monday.*

Let $s_1 s_2 s_3 s_4 s_5 s_6 s_7 s_8 s_9$ **denote your student number (i.e. the digits of your student number are, in order,** $s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_8, s_9$**).**

1 (a) Give an example of a vector space V over the field $\mathbb{Q}$, but choose it in such a way that no other student has used the same V as their answer to this question.

(b) Give an example of an (s_2+s_3)-dimensional vector space V over the field $\mathbb{C}$, and write down a basis for V.

(c) Give a list of s_9+3 vectors which span a vector space (of your choosing) but do not form a basis for this vector space.

(d) State whether or not the three vectors

$$\begin{bmatrix} s_1 \\ i s_2 \\ s_3 \end{bmatrix}, \begin{bmatrix} i s_4 \\ s_5 \\ s_6 \end{bmatrix}, \begin{bmatrix} s_7 \\ s_8 \\ i s_9 \end{bmatrix}$$

are linearly independent in $\mathbb{C}^3$ (where i denotes the square root of -1).

(e) Is the set

$$U = \left\{ \begin{bmatrix} (2s_2 - s_3)a \\ 3a^{s_7+1} \\ (s_6+s_8)a \end{bmatrix} : a \in \mathbb{R} \right\}$$

a subspace of $\mathbb{R}^3$? Justify your answer.

2 (a) Show that the list

$$B = \left(\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}, \begin{bmatrix} 9 \\ -11 \\ -5 \end{bmatrix} \right)$$

is a basis for the vector space $V = \mathbb{R}^3$ over the field $\mathbb{R}$, by first showing that the three vectors are linearly independent, and secondly showing that they are spanning.

(b) Decide whether or not the list

$$B' = \left(\begin{bmatrix} s_1 \\ s_2 \\ s_3 \end{bmatrix}, \begin{bmatrix} s_4 \\ s_5 \\ s_6 \end{bmatrix}, \begin{bmatrix} s_7 \\ s_8 \\ s_9 \end{bmatrix} \right)$$

is linearly independent. Is B' spanning? Is B' a basis for $\mathbb{R}^3$? Justify your answers.

(c) For each vector in B', determine its coordinate representation with respect to the basis B.

(d) If your list B' is a basis, find the transition matrix $P = P_{B,B'}$ from B to B' (hint: use your answer to (c)). [If your list B' is *not* a basis then just ignore this part of the question - you will automatically score full marks!]

3 State whether the following are True or False, giving reasons.

(a) The list

$$B = \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 7 \end{bmatrix} \right)$$

is a basis for the vector space $V = \mathbb{R}^3$ over the field $\mathbb{R}$.

(b) The list

$$B = \left(\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right)$$

is a basis for the vector space $V = \mathbb{C}^3$ over the field $\mathbb{C}$.

(c) The list

$$B = \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \begin{bmatrix} i \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ i \end{bmatrix} \right)$$

is a basis for the vector space $V = \mathbb{C}^2$ over the field $\mathbb{C}$. [Here i denotes the complex number $\sqrt{-1}$].

(d) The set

$$U = \left\{ \begin{bmatrix} t \\ t^2 \end{bmatrix} : t \in \mathbb{R} \right\}$$

is a vector subspace of $\mathbb{R}^2$.

(e) The set

$$U = \left\{ \begin{bmatrix} 0 \\ s \\ s+t \end{bmatrix} : s,t \in \mathbb{R} \right\}$$

is a vector subspace of $\mathbb{R}^3$.

4 State whether the following are True or False, giving brief reasons.

(a) Every finite-dimensional vector space contains only finitely many vectors.

(b) Other than the trivial vector space $\{0\}$, every vector space contains infinitely many vectors.

(c) If U_1 and U_2 are finite-dimensional subspaces of a vector space V, then we have $\dim(U_1 + U_2) = \dim(U_1) + \dim(U_2)$.

(d) If U_1 and U_2 are subspaces of a vector space V, then $u_1 - u_2 \in U_1 + U_2$ for every $u_1 \in U_1$, $u_2 \in U_2$.

5 Let V be a finite-dimensional vector space, and let U be a subspace of V satisfying $\dim(U) = \dim(V)$.

Prove that $U = V$.

6 Let $v_1, v_2, \ldots, v_n$ be a list of vectors in a vector space V, and suppose that these vectors are spanning. Show that we can obtain a basis for V by deleting some vectors from this list.

Show that the vectors

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 3 \\ 4 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

span $\mathbb{R}^3$, and find a sublist which is a basis.