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Ergodic optimization and prevalence

Abstract:
Ergodic optimization is the process of finding invariant probability measures that maximize the integrals of a given function. This topic is naturally motivated by the studies in controlling chaos and bifurcation to riddled basins of chaotic attraction. In this talk, we will particularly concentrate on the long-standing periodic optimization problem in this area.

To be more precise, given a dynamical system, we say that a performance function has property P if its time averages along orbits are maximized at a periodic orbit. It is conjectured by several authors that for sufficiently hyperbolic dynamical systems, property P should be typical among sufficiently regular performance functions. In contrast to the literature, by considering typicality in the topological sense, we first address this problem using a probabilistic notion of typicality that is suitable to infinite dimension: the concept of prevalence as introduced by Hunt, Sauer, and Yorke. For the one-sided shift on two symbols, we prove that property P is prevalent in spaces of functions with a strong modulus of regularity. Our proof uses Haar wavelets to approximate the ergodic optimization problem by a finite-dimensional one, which can be conveniently restated as a maximum cycle mean problem (a fundamental problem in combinatorial optimization) on a de Bruijin graph.