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How to solve Falconer's distance problem using ergodic theory

Abstract:
Given a set $E \in \mathbb{R}^d$, the distance set of E is defined by $D(E) = \{|x-y| : x, y \in E\}$, i.e. the set of all distances realised by pairs of points in E. Inspired by the famous Erdős distinct distances problem (recently resolved by Guth and Katz), Falconer's distance problem attempts to relate the size of $D(E)$ with the size of E, where E is assumed to be uncountable and 'size' means Hausdorff dimension. One conjecture is the following: if the Hausdorff dimension of E is greater than or equal to $d/2$, then the distance set of E should have maximal Hausdorff dimension. There are numerous partial results available, but the full conjecture is still wide open. A recent innovation of Tuomas Orponen was that the pioneering work of Hochman and Shmerkin on projections (and in particular C^1 images) of fractals could be applied to the distance problem, at least in the plane. Together with Andy Fergsuon and Tuomas Sahlsten, we developed this idea to give a sufficient condition for the conjecture to be true in the plane based on the ergodic theory of the scenery flow of measures supported on E. I will discuss this result and give some concrete applications. I will also mention recent joint work with Mark Pollicott where we consider how to extend this approach to deal with certain conformally generated fractals like Julia sets or limit sets of Kleinian groups.