Toric Geometry (L24)

Navid Nabijou

Toric varieties form a special and important class of algebraic varieties. They are characterised by the presence of a strong torus symmetry, or equivalently by their defining equations being binomial. This additional structure strongly constrains the geometry, to the extent that it can be captured entirely by combinatorics. As a result, problems that are difficult or impossible to solve for arbitrary varieties often have explicit and elegant solutions in the toric setting.

The principal goal of this course is for you to develop intuition, fluency and confidence in dealing with algebraic varieties. Toric geometry is the perfect vehicle for this: it provides a varied roster of interesting examples, which can be studied more or less explicitly.

Emphasising examples throughout, we will cover a range of topics (items marked ($\star$) will be included if time permits):

- Definition of toric varieties: binomial equations, monoidal rings, torus actions.
- Combinatorial shadows: cones and fans. Orbit-cone correspondence. Toric morphisms.
- Complete and projective toric varieties. Smoothness.
- Toric surfaces: singularities and global structure.
- Subdivisions, blowups, resolution of singularities.
- Divisors: Weil and Cartier. Rational equivalence.
- Singular cohomology of toric varieties.
- ($\star$) Quotient constructions and toric GIT.
- ($\star$) Flat and reduced morphisms. Semistable reduction.

Prerequisites

Part II Algebraic Geometry is a necessity, as is as a firm grasp of rings and ideals, so be sure to brush up on Part IB Groups, Rings and Modules.

Part III Algebraic Geometry is very strongly recommended. That course and this one are complementary: the first develops a magnificent theory, whereas the second puts that theory into practice, in a setting where (almost) everything can be explicitly computed.

Preliminary Reading

1. I. Shafarevich. *Basic algebraic geometry. 1. Varieties in Projective Space.* Third edition. Translated from the 2007 third Russian edition. Springer, Heidelberg, 2013.
2. M. Reid. *Undergraduate Algebraic Geometry.* London Mathematical Society Student Texts, 12. Cambridge University Press, Cambridge, 1988.
3. M. F. Atiyah and I. G. Macdonald. *Introduction to Commutative Algebra* Addison–Wesley Publishing Co., Reading, Mass.–London-Don Mills, Ont. 1969.

Literature

1. W. Fulton. *Introduction to Toric Varieties.*
   Annals of Mathematics Studies, 131. The William H. Roever Lectures in Geometry. Princeton University Press, Princeton, NJ, 1993.

2. D. Cox, J. B. Little and H. K. Schenck. *Toric Varieties.*
   Graduate Studies in Mathematics, 124. American Mathematical Society, Providence, RI, 2011.

Additional support

Four examples sheets will be provided and four associated examples classes will be given. There will be a one-hour revision class in the Easter Term.