1. Moduli of Curves

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Moduli spaces are ubiquitous in algebraic geometry. They arise from attempts to parametrise some given class of geometric objects — for example, subschemes of projective space, or line bundles on a fixed algebraic curve. Miraculously, the set of such geometric objects is itself endowed with a natural scheme structure, reflecting how the parametrised objects relate to one another. This output is referred to as a moduli space.

Perhaps the most important example is the moduli space of algebraic curves. Though the objects being parametrised are relatively simple, the resulting moduli space is extremely intricate, and remains an active topic of research, with many open questions.

This essay will focus on the moduli space of smooth pointed curves $\mathcal{M}_{g,n}$, and the compactification given by the moduli space of stable curves $\overline{\mathcal{M}}_{g,n}$. The emphasis will be on explicit examples and computations, and not on abstract generalities. As such, you will take the existence of the moduli space on faith, and focus instead on its behaviour.

There are countless interesting aspects to explore. You will focus on one of the following topics:

- Stable and semistable reduction.
- Intersection theory: psi, kappa and lambda classes.
- Hurwitz schemes and admissible covers.
- Topology: Euler characteristic, top-weight cohomology.
- Variants: Hassett spaces, wonderful models.

Which one you choose will depend on your specific strengths and inclinations. I am also open to considering additional suggestions.

Even a very good essay is not expected to contain original research. However, you will have the opportunity to demonstrate deep understanding, by providing detailed treatments of examples not covered in the existing literature.

Relevant Courses

Essential: Part III Algebraic Geometry, Part II Algebraic Geometry.

Useful: Part II Riemann Surfaces, Part III Commutative Algebra, Part III Algebraic Topology.

References

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[3] Pandharipande, R. *A calculus for the moduli space of curves.*
Algebraic geometry: Salt Lake City 2015, 459–487, Proc. Sympos. Pure Math., 97.1, Amer. Math. Soc., Providence, RI, 2018

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