Two days of Algorithms at Queen Mary, University of London



# Martin Dyer Day and Queen Mary Algorithms Day (QMAD)

Monday 16th and Tuesday 17th July 2018

We are pleased to present two days of Algorithms and Computational Complexity at Queen Mary, University of London. On Monday we celebrate the career of Martin Dyer, with a programme of talks by friends and collaborators. On Tuesday, there is an Algorithms Day, following the no-frills model that has been established in a sequence of meetings at several sites in the UK, including Bristol, Liverpool, Middlesex, Oxford and Warwick.

Colin Cooper, Leslie Ann Goldberg and Mark Jerrum

## **Martin Dyer Day**

### Monday 16th July 2018

Please note that talks are in the ArtsTwo Lecture Theatre, and refreshments and lunch are in the Foyer of the same building.

#### Programme

09:45-10:25 Coffee

- 10:25-10:30 Welcome.
- 10:30–11:05 Jin-Yi Cai, University of Wisconsin, Madison, Classification for counting problems
- 11:10–11:45 Alan Frieze, Carnegie Mellon University, Pittsburgh, Coloring (random) hypergraphs
- **11:50–12:25** Leen Stougie, Centrum Wiskunde en Informatica, Amsterdam, A decomposition theory for vertex enumeration of convex polyhedra

12:30-13:30 Lunch

- 13:30–14:05 Catherine Greenhill, University of New South Wales, Sampling graphs in at least two ways
- **14:10–14:45** Colin McDiarmid, University of Oxford, On the modularity of random graphs G(n, p)

14:50–15:25 Ravi Kannan, Microsoft Research, India, Spectral rounding

15:25–15:55 Tea break

**15:55–16:30** Haiko Müller, University of Leeds, *The switch chain on perfect matchings and the recognition of quasi-monotone graphs* 

16:35–17:10 Eric Vigoda, Georgia Tech, Path coupling and belief propagation

17:10 Reception.

This day is organised by Colin Cooper (King's College London), Leslie Ann Goldberg (University of Oxford) and Mark Jerrum (Queen Mary, University of London).

## **Queen Mary Algorithms Day (QMAD)**

### **Tuesday 17th July 2018**

Please note that talks are in the ArtsTwo Lecture Theatre, and refreshments and lunch are in the Foyer of the same building.

#### Programme

- **10:00–10:40** Holger Dell, Universität des Saarlandes, *How to detect and count small subgraphs efficiently*
- 10:40-11:10 Coffee
- **11:10–11:50** Miriam Backens, University of Oxford, *Holant problems and quantum information theory*
- **11:55–12:35** Heng Guo, University of Edinburgh, A polynomial-time approximation algorithm for all-terminal network reliability

12:40-13:40 Lunch

- **13:40–14:20** Viresh Patel, University of Amsterdam, Zero-free regions and approximation algorithms for graph polynomials
- 14:25–15:05 Andreas Galanis University of Oxford, Inapproximability of the independent set polynomial in the complex plane

15:05–15:35 Tea break.

- 15:35–16:15 John Fearnley, University of Liverpool, End of Potential Line
- **16:20–17:00** László Végh, London School of Economics, *A simpler and faster strongly polynomial algorithm for generalized flow maximization*

This day is organised by Mark Jerrum (Queen Mary, University of London).

## Abstracts

#### Miriam Backens, Holant problems and quantum information theory.

ABSTRACT. Holant problems are a family of counting problems on graphs, parameterised by sets of complex-valued functions of Boolean inputs. While the Holant framework was originally inspired by ideas from quantum computation, this connection has not been exploited before to analyse their complexity. We show how to apply ideas from quantum information theory to the analysis of Holant problems, using the bijection that exists between *n*-ary complex-valued functions of Boolean inputs and states of qubits (i.e. quantum bits), which are described by vectors in  $\mathbb{C}^{2^n}$ . Many interesting families of functions in the Holant framework correspond to families of quantum states that are of independent interest in quantum information theory. We sketch the role these ideas played in the derivation of two new Holant dichotomies, which make use of methods and knowledge from the theory of quantum entanglement. One of the new results, the full dichotomy for Holant<sup>c</sup>, solves a long-standing open problem.

#### Jin-Yi Cai, Classification for Counting Problems.

ABSTRACT. Suppose  $\alpha$  and  $\beta$  are two angles satisfying  $\tan(\alpha) = 2\tan(\beta) > 0$ . Can both  $\alpha$  and  $\beta$  be rational multiples of  $\pi$ ?

We encounter questions like this in the classification of counting problems, an area Martin Dyer had pioneered and led for many years. In this talk I will outline a proof to the specific question above on the incommensurability of angles and tangent values using character theory, and describe how its answer leads to a complexity dichotomy theorem. Then I will give an overview of the status of the classification program, highlighting some of the seminal contributions by Martin and his colleagues.

#### Holger Dell, How to detect and count small subgraphs efficiently.

ABSTRACT. This talk will survey algorithms for the subgraph isomorphism problem from the viewpoint of fine-grained parameterized complexity. Given two graphs H and G, what's the fastest algorithm to detect a subgraph of G that's isomorphic to H? How fast can we (exactly or approximately) count these occurrences? What are important open questions in this area?

#### John Fearnley, End of Potential Line.

ABSTRACT. We introduce the problem EndOfPotentialLine and the corresponding complexity class EOPL of all problems that can be reduced to it in polynomial time. This class captures problems that admit a single combinatorial proof of their joint membership in the complexity classes PPAD of fixpoint problems and PLS of local search problems. Our two main results are to show that both PL-Contraction (Piecewise-Linear Contraction, defined with a linear FIXP circuit) and P-LCP are in EOPL. Our reductions imply that the promise versions of PL-Contraction and P-LCP are in the promise class UniqueEOPL, which corresponds to the case of a single potential line. This also shows that simple-stochastic, discounted, mean-payoff, and parity games are in EOPL.

#### Alan Frieze, Coloring (random) hypergraphs.

ABSTRACT. We sketch some results related to coloring hypergraphs.

- 1. An MCMC algorithm for the case where the maximum degree is  $\Omega(\log n)$ .
- 2. Connectivity of the set of random colorings of a random hypergraph.
- 3. The Game Chromatic Number of random hypergraphs.

1 and 2 are joint with Michael Anastos. 3 is joint with Debsoumya Chakrabati and Mihir Hasbani

#### Andreas Galanis, Inapproximability of the independent set polynomial in the complex plane.

ABSTRACT. For a graph G, let  $p_G(\lambda)$  denote the independent set polynomial of G with parameter  $\lambda$ . For which  $\lambda$  can we approximate the value  $p_G(\lambda)$  on graphs G of maximum degree  $\Delta$ ? This problem is already well understood when  $\lambda$  is a real number using connections to phase transitions in statistical physics on the  $\Delta$ -regular tree.

Understanding the case where  $\lambda$  is complex turns out to be more challenging. Peters and Regts studied an analogue of the phase transition on the  $\Delta$ -regular tree for general complex values of  $\lambda$ . They identified a region  $\Lambda(\Delta)$  in the complex plane and, motivated by the real case, they asked whether  $\Lambda(\Delta)$  marks the approximability threshold for general complex values  $\lambda$ .

We show that for every  $\lambda$  outside of  $\Lambda(\Delta)$ , the problem of approximating  $p_G(\lambda)$  on graphs G with max degree  $\Delta$  is indeed NP-hard. In fact, when  $\lambda$  is outside of the region and is not a positive real number, we give the stronger result that approximating  $p_G(\lambda)$  is actually #P-hard.

This is joint work with Ivona Bezáková, Leslie Ann Goldberg, and Daniel Štefankovič.

#### **Catherine Greenhill**, *Sampling graphs in at least two ways*.

ABSTRACT. It is well known that sampling and counting are very closely related, and this has been utilised in algorithm design in many ways. I will discuss (at least) two approaches to sampling graphs which make use of methods or insights from asymptotic enumeration.

#### Heng Guo, A polynomial-time approximation algorithm for all-terminal network reliability.

ABSTRACT. All-terminal network reliability is the probability that, in a undirected graph, assuming each edge fails independently, the remaining graph is still connected. We will present a fully polynomial-time randomised approximation scheme (FPRAS) for this problem. Our main contribution is to confirm a conjecture by Gorodezky and Pak (Random Struct. Algorithms, 2014), that the expected running time of the "cluster-popping" algorithm in bi-directed graphs is bounded by a polynomial in the size of the input.

Joint work with Mark Jerrum (QMUL).

#### Ravi Kannan, Spectral rounding.

ABSTRACT. We consider a class of 0-1 optimization problems which includes Max-Cut, Maxbalanced cut, 2-clustering, Maximum Margin Clustering among others:

 $\min x^T A x$  + (a convex function of a constant number of linear forms in x), subject to  $x_i$  are 0 or 1.

Our algorithm is to replace A by a low rank approximation of it, solve the corresponding quadratic program over the reals and then use an auxiliary linear program to find a near-integer solution. We show an additive error bound which depends on the spectrum of A. Certain classes of problems where the bounds are good will be discussed.

#### **Colin McDiarmid**, On the modularity of random graphs G(n, p).

ABSTRACT. For a given graph G, each vertex partition has a modularity score, with higher values taken to indicate that the partition better captures community structure in G. The modularity  $q^*(G)$  (where  $0 \le q^*(G) \le 1$ ) of the graph G is defined to be the maximum over all vertex partitions of the modularity score. Many heuristics used in community detection are based on finding partitions with high modularity score.

For the Erdős-Rényi random graph G(n, p) with n vertices and edge-probability p, the likely modularity has three phases. For  $np \leq 1 + o(1)$  the modularity is 1 + o(1) with high probability (whp); and for  $np \to \infty$  the modularity is o(1) whp. For np bounded above and bounded above 1, the modularity is bounded away from 0 and 1 whp. Further, for  $np \geq 1$  (and p not approaching 1 very quickly) whp  $q^*(G(n, p))$  has order  $(np)^{-1/2}$ , in accord with a conjecture by Reichardt and Bornholdt in 2006 (and disproving another conjecture from the physics literature). We shall discuss an algorithm showing that  $q^*(G(n, p))$  has at least this order.

This is joint work with Fiona Skerman.

# **Haiko Müller**, The switch chain on perfect matchings and the recognition of quasi-monotone graphs.

ABSTRACT. We consider the switch Markov chain of Diaconis, Graham and Holmes to approximately count the perfect matchings in graphs from hereditary classes. We determine the largest hereditary class of bipartite graphs for which the chain is ergodic, and define a new hereditary class of graphs for which it is rapidly mixing, the quasi-monotone graphs. We investigate the structure of quasi-monotone graphs and construct a polynomial time recognition algorithm for graphs in this class.

#### Viresh Patel, Zero-free regions and approximation algorithms for graph polynomials.

ABSTRACT. In this talk, I discuss a new way of constructing deterministic polynomial-time approximation algorithms for computing complex-valued evaluations of a large class of graph polynomials on bounded degree graphs. In particular, the approach works for the Tutte polynomial and independence polynomial, as well as partition functions of complex-valued spin and edge-coloring models. This builds on a recent work of Barvinok, which provides a new algorithmic approach besides the existing Markov chain Monte Carlo method and the correlation decay method for these types of problems. (Joint work with Guus Regts.)

#### Leen Stougie, A decomposition theory for vertex enumeration of convex polyhedra.

ABSTRACT. In the last years the vertex enumeration problem of polyhedra has seen a revival in the study of metabolic networks, which increased the demand for efficient vertex enumeration algorithms for high-dimensional polyhedra given by equalities and inequalities. The complexity of enumeration of vertices of polytopes (bounded polyhedral) is a famous open problem in discrete and computational geometry.

In this lecture we do not solve this problem, but present a type of fixed parameter tractable result. We apply the concept of branch-decomposition to the vertex enumeration problem of polyhedra  $P = \{x : Ax = b; x \ge 0\}$ . For that purpose, we introduce the concept of k-module and show how it relates to the separators of the linear matroid generated by the columns of A. This then translates structural properties of the matroidal branch-decomposition to the context of polyhedra. We then use this to present a total polynomial time algorithm for enumerating all vertices of polytopes P for which the branch-width of the linear matroid generated by S is bounded by a constant k. If time permits we will show a truly fixed parameter tractable result, with time being an exponential function of the parameters only times a polynomial function of the input/output size, if on top of bounded branchwidth we also assume that all square submatrices of A have bounded determinant.

This paper is joint work with Arne Reimers.

#### László Végh, A simpler and faster strongly polynomial algorithm for generalized flow maximization.

ABSTRACT. Generalized flows are a classical extension of network flows, where the flow gets multiplied by a certain gain or loss factor while traversing an arc. This is a widely applicable model,

as the factors can be used to model physical changes such as leakage or theft; or alternatively, they can represent conversions between different types of entities, e.g. different currencies. In the talk, I present a new strongly polynomial algorithm for generalized flow maximization. Besides improving on the running time of the previous strongly polynomial algorithm by a factor  $O(n^2)$ , the algorithm is also substantially simpler. This is joint work with Neil Olver (VU Amsterdam).

#### Eric Vigoda, Path coupling and belief propagation.

ABSTRACT. One of Martin Dyer's most important contributions was the introduction of the path coupling approach, with his student Russ Bubley. This technique enabled many improved rapid mixing results, especially for Markov chains for randomly sampling colorings and independent sets. We'll look at connections between belief propagation (BP) algorithms and path coupling for the analysis of Markov chains for sampling independent sets in the hard-core model.

## Martin Dyer: a brief bio



Image courtesy of Catherine Greenhill

Martin Dyer graduated from the University of Leeds in 1967, obtained his MSc from Imperial College London in 1968 and his PhD from the University of Leeds in 1979. His research interests lie in theoretical computer science, discrete optimisation and combinatorics. Currently, he focuses on the complexity of counting and the efficiency of Markov chain algorithms for approximate counting.

With Alan Frieze and Ravi Kannan, he presented the first algorithm for approximating the volume of a convex body whose runtime scales as a polynomial in the number of dimensions. This breakthrough result<sup>1</sup> initiated a very active and fruitful line of research, and has been heavily cited. For this article, the authors were awarded the Fulkerson Prize in Discrete Mathematics by the American Mathematical Society and the Mathematical Programming Society. With Russ Bubley, he introduced the path coupling method for bounding the mixing time of Markov chains.<sup>2</sup> Path coupling has proved a valuable tool in the analysis of mixing times, and is now part of the toolkit of everyone working in the area. With various authors he has made several important contributions<sup>3,4</sup> to the study of the complexity of counting constraint satisfaction problems.

At the 2013 International Colloquium on Automata, Languages and Programming (ICALP), in Riga, Martin Dyer received the *EATCS Award*, which acknowledges "extensive and widely recognised contributions to theoretical computer science over a life long scientific career".

<sup>&</sup>lt;sup>1</sup>Martin Dyer, Alan Frieze and Ravi Kannan, A random polynomial-time algorithm for approximating the volume of convex bodies. *J. Assoc. Comput. Mach.* **38** (1991), no. 1, 1–17.

<sup>&</sup>lt;sup>2</sup>Russ Bubley and Martin Dyer, Path coupling: a technique for proving rapid mixing in Markov chains, *Proceedings of the 38th Annual Symposium on Foundations of Computer Science, IEEE* (1997), 223–231.

<sup>&</sup>lt;sup>3</sup>Martin Dyer, Catherine Greenhill, The complexity of counting graph homomorphisms, *Random Structures Algorithms* **17** (2000), no. 3–4, 260–289.

<sup>&</sup>lt;sup>4</sup>Martin Dyer, David Richerby, An effective dichotomy for the counting constraint satisfaction problem, *SIAM J. Comput.* **42** (2013), no. 3, 1245–1274.

## About the host

Queen Mary, University of London traces its origins back to the late 19th century with the establishment of the People's Palace, a philanthropic organisation designed to bring culture, recreation and education to the East End of London. Today it is one of the UK's leading research-focused universities. With over 20,000 students, it is amongst the largest of the colleges of the University of London. Queen Mary's 4,000 staff teach and research across a wide range of subjects in Science and Engineering, the Humanities, Social Sciences and Laws, and Medicine and Dentistry. The School of Mathematical Sciences has a strong research presence across the spectrum of areas within Pure and Applied Mathematics, and is currently organised into six research groups, namely: Algebra, Combinatorics, Complex Systems and Networks, Dynamical Systems and Statistical Physics, Geometry and Analysis, and Probability and Applications. The School also has large and popular undergraduate and graduate programmes. The School is entering an exciting period of growth with several new posts across a broad range of disciplines. The School's building is being completely transformed to provide state-of-the-art research, teaching and study facilities for staff and students.

## **Practicalities**

- *Internet access*. Free WiFi is available on campus. The name of the network is QM-Events. The password for the network 16th 17th July is ZmhH5152. Also, for those who have access to eduroam and have already configured access at their home institution, eduroam is available anywhere on campus where WiFi is available.
- *Breakout space*. ArtsTwo room 2.18 (capacity 18) has been booked as a breakout room on *Monday 16th July only*.
- *Assistance*. If you require assistance during the conference, the Conference Reception desk is open in the foyer of the ArtsTwo Building at the following times:
  - Monday 16th July, 09:00-17:30,
  - Tuesday 17th July, 09:00–16:30.

You can contact the Organising Committee at qmadlondon@gmail.com with any queries.

• *Health and Safety.* If a fire alarm sounds, please evacuate the building and immediately proceed to the nearest assembly point which are as follows:

ArtsTwo Building Westfield Way (Assembly Point D)

There are no scheduled fire alarm tests due to be conducted on Monday 16th or Tuesday 17th.

Security Officers operate on our Mile End Campus. Their control room is on the Ground Floor, France House. You can contact them on 020 7882 5000 or internal extension 5000. In case of emergency, please dial 020 7882 3333 (internal extension 3333).

Smoking is only permitted at designated smoking areas/shelters (indicated on the map on the next page).

# **Campus map**



Mile End Ca	du	<b>US</b> Residential	Facilities		i) Information
ArtsOne	37	Albert Stern Cottages	Advice and Counselling Service	27 Vi	isitors who require further
ArtsTwo	35	Albert Stern House	Bookshop 🕮	22 II	formation or assistance should
Arts Research Centre	39	Beaumont Court 53	Canalside	03 03	lease go to the main reception the Otteens' Building
Bancroft Building	31	Chapman House 43	Careers Centre	19	
Bancroft Road		Chesney House 45	Clock Tower	20	I he smoking of cigarettes or tobacco products are only
Teaching Rooms	10	Creed Court 57	CopyShop	56	permitted at designated smoking
Peter Landin Building	0	France House 55	The Curve (R)	47	areas / shelters indicated on this
(computer science)		Feilden House 46	Disability and Dyslexia Service	31	map.
Engineering Bullaing	0	Hatton House 40	Draners' Bar and Kitchen	α	Electronic cigarettes permitted
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G.O. Jones Building	25	Lindon House 21	Ground Cate	5	C These premises are alarmed
Geography	26		The Nest	24	and monitored by CCTV:
Graduate Centre	18		Housing Hub	48	please call Security on
Informatics Teaching		ryinen rouse 33	IT Services	19	+44 (0)20 7882 5000
Laboratories	5	Maurice Court	Mucci's (R)	29	for more information.
Joseph Priestley Building	41	Maynard House 44	Occupational Health Service/	Y	ey
Library (B)	32	Pooley House 60	Student Health Service	28	1 ihmer/hoolichen
Law	36	Selincourt House 51	Octagon	1 <u>9a</u>	
Lock-keeper's Cottage	42	Varey House 49	Portering and Postal Services	1	) Fitness centre
Occupational Health	1		Qmotion Health and Fitness Cen	itre	A) Refreshment: Bar/Eatery/Coffee place
Donnla's Dalana/Great Hall	4				Staff car park
			Santander Bank	2	
Queens' Building U	19		Security	38/54	Bicycle parking
Scape Building	64		St Benet's Chaplaincy	23	Bicycle lockers
Temporary Building	61		Student Enquiry Centre	19	Cash machine
			Students' Union Hub 🏟	34	Smoking area / shelter
Building construction site	14		Union Shop (R)	6	
Building closed for			Village Shop	52	
major refurbishment	4		Westfield Nursery	11	