Holant problems and quantum information theory

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Outline

Introduction to holant problems

Basic quantum information theory

The new results

Conclusions

Outline

(Boolean) holant problems

Fix a set of functions $\mathcal{F}$.

Throughout, functions take Boolean inputs and yield algebraic complex outputs.

Name $\textsc{Holant}(\mathcal{F})$

Instance a signature grid $\Omega = (G, \mathcal{F}, \pi)$, where

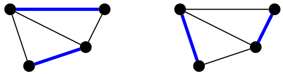
- $G = (V, E)$ is a finite multigraph
- $\pi : V \to \mathcal{F} :: v \mapsto f_v$ is a map that furthermore creates a bijection between $E(v)$ and the arguments of f_v

Output the number

$$\text{Holant}_\Omega := \sum_{\sigma : E \to \{0,1\}} \prod_{v \in V} f_v \left(\sigma|_{E(v)} \right)$$

Problems expressible in the holant framework

counting perfect matchings:



$$\sum_{\sigma:E\to\{0,1\}}\prod_{v\in V}\mathrm{ONE}_{\deg(v)}(\sigma|_{E(v)})$$

where $\mathrm{ONE}_n(x_1,\ldots,x_n) = \begin{cases} 1 & \text{if } \sum_{k=1}^{n} x_k = 1 \\ 0 & \text{otherwise} \end{cases}$

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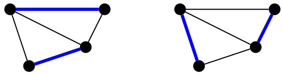

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also:

- many other counting problems defined on graphs
- counting graph homomorphisms
- counting constraint satisfaction problems
- partition functions from statistical physics
- classical simulation of quantum computations
- ...

Counting complexity dichotomies

Complexity classes

- FP: counting problems that can be solved in polynomial time
- $\#$P: counting complexity equivalent of NP

Unless FP is equal to $\#$P, there exist intermediate problems in $\#\text{P} \setminus \text{FP}$, which are not $\#$P-complete (by a variant of Ladner's theorem).

In the holant framework, all existing complexity classifications are dichotomies, partitioning families of functions into

- those that are polynomial-time solvable, and
- those that are $\#$P-hard

(without any $\#$P-intermediate problems)

Holant families and existing results

No full complexity classification for $\textsc{Holant}(\mathcal{F})$ yet

Allowing additional unary functions:

- $\textsc{Holant}^*(\mathcal{F}) := \textsc{Holant}(\mathcal{F} \cup \{\text{all unaries}\})$ [Cai, Lu, Xia 2011]
- $\textsc{Holant}^+(\mathcal{F}) := \textsc{Holant}(\mathcal{F} \cup \{\delta_0, \delta_1, \delta_+, \delta_-\})$ [here]
- $\textsc{Holant}^c(\mathcal{F}) := \textsc{Holant}(\mathcal{F} \cup \{\delta_0, \delta_1\})$ [here]

where

$$\begin{cases} \delta_0(0) = 1 \\ \delta_0(1) = 0, \end{cases} \quad \begin{cases} \delta_1(0) = 0 \\ \delta_1(1) = 1, \end{cases} \quad \begin{cases} \delta_+(0) = 1 \\ \delta_+(1) = 1, \end{cases} \quad \text{and} \quad \begin{cases} \delta_-(0) = 1 \\ \delta_-(1) = -1. \end{cases}$$

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Allowing additional equality functions:

- $\#\text{CSP}(\mathcal{F}) = \text{HOLANT}(\mathcal{F} \cup \{\text{EQ}_k \mid k \in \mathbb{N}\})$ [Cai, Lu, Xia 2009]
- $\#\text{CSP}_2^c(\mathcal{F}) := \text{HOLANT}(\mathcal{F} \cup \{\delta_0, \delta_1\} \cup \{\text{EQ}_{2k} \mid k \in \mathbb{N}\})$ [Cai, Lu, Xia 2017]

Holographic transformations

There is a bijection between functions and lists of their values

$$f : \{0,1\}^n \to \mathbb{C} \qquad \leftrightarrow \qquad \mathbf{f} \in \mathbb{C}^{2^n}$$

Example: $g(x,y) = 2x + y \quad \leftrightarrow \quad \mathbf{g} = (0,1,2,3)^T$

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Example: If $M = \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$, then $(M \circ g)(x,y) = 3 - 4x - 5y + 6xy$, because

$$(M \otimes M)\mathbf{g} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 3 \\ -2 \\ -1 \\ 0 \end{pmatrix}$$

Holographic transformations and complexity

Valiant's holant theorem

Suppose $\mathcal{F}, \mathcal{G}$ are sets of functions and suppose M is an invertible 2×2 matrix. Then

$$\textsc{Holant}\,(\mathcal{F} \mid \mathcal{G}) \equiv_T \textsc{Holant}\,\left(M \circ \mathcal{F} \mid (M^T)^{-1} \circ \mathcal{G}\right).$$

If O is a 2×2 orthogonal matrix, then

$$\textsc{Holant}\,(\mathcal{F}) \equiv_T \textsc{Holant}\,(O \circ \mathcal{F}).$$

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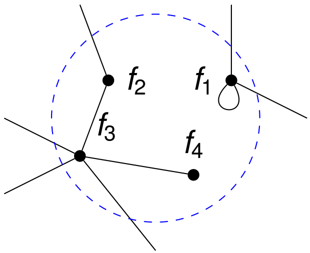
$$\textsc{Holant}\,(\mathcal{F}) \equiv_T \textsc{Holant}\,(O \circ \mathcal{F}).$$

Example: As $M = \left(\begin{smallmatrix} 0 & -1 \\ 1 & 0 \end{smallmatrix}\right)$ is orthogonal,

$$\textsc{Holant}\,(\{g, \mathrm{EQ}_3\}) \equiv_T \textsc{Holant}\,(\{M \circ g, M \circ \mathrm{EQ}_3\}),$$

where $g(x, y) = 2x + y$ and $(M \circ g)(x, y) = 3 - 4x - 5y + 6xy$.

Holant gadgets



for any $\mathbf{x} : E_{\text{ext}} \to \{0, 1\}$:

$$g(\mathbf{x}) = \sum_{\mathbf{y}:E_{\text{int}}\to\{0,1\}} \prod_{v\in V} f_v(\mathbf{x}, \mathbf{y}|_{E(v)})$$

Let $\langle\mathcal{F}\rangle$ be the closure of $\mathcal{F}$ under taking holant gadgets, then

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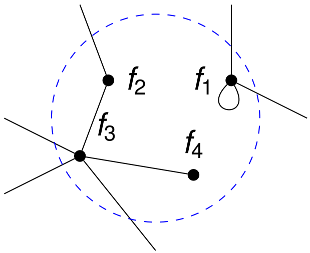

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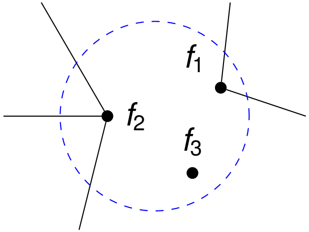
Let $\langle \mathcal{F} \rangle$ be the closure of $\mathcal{F}$ under taking holant gadgets, then

$$\textsc{Holant}\,(\mathcal{F}) \equiv_T \textsc{Holant}\,(\langle \mathcal{F} \rangle)$$

A holant gadget with zero internal edges is called a tensor product



Tractable families, part 1

HOLANT $(\mathcal{F})$ can be solved in polynomial time in the following cases:

- unary and binary functions: $\mathcal{F} \subseteq \langle\{f \mid \text{arity}(f) = 1 \text{ or } \text{arity}(f) = 2\}\rangle$
- generalised equality functions: there exists M satisfying $M^T M \in \left\{\left(\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right), \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)\right\}$, such that

$$\mathcal{F} \subseteq \left\langle M \circ \left\{f \,\middle|\, \exists \mathbf{a} \in \{0,1\}^{\text{arity}(f)} \text{ s.t. } \forall \mathbf{x} \notin \{\mathbf{a}, \bar{\mathbf{a}}\}, f(\mathbf{x}) = 0\right\}\right\rangle$$

- generalised matching functions: there exists M satisfying $M^T M = \left(\begin{smallmatrix} 0 & 1 \\ 1 & 0 \end{smallmatrix}\right)$, such that

$$\mathcal{F} \subseteq \langle M \circ \{f \mid f(\mathbf{x}) = 0 \text{ unless } |\mathbf{x}| \leq 1\}\rangle$$

[Cai, Lu, Xia 2011]

Tractable families, part 2

Let $\mathcal{A}$ be the set of affine functions

$$f(\mathbf{x}) = c\, \chi_{A\mathbf{x}=\mathbf{b}}\, i^{l(\mathbf{x})}\, (-1)^{q(\mathbf{x})}$$

HOLANT $(\mathcal{F})$ can be solved in polynomial time in the following cases:

- transformable to affine: there exists invertible M satisfying

$$M^T \circ \{\mathrm{EQ}_2, \delta_0, \delta_1\} \subseteq \mathcal{A} \quad \text{and} \quad \mathcal{F} \subseteq M \circ \mathcal{A}$$

[Cai, Huang, Lu 2012]

- local affine: $\mathcal{F}$ is a subset of

$$\bigcup_{n \in \mathbb{N}} \{f \mid \text{ if } f(x_1, \ldots, x_n) \neq 0 \text{ then } (T^{x_1} \otimes \ldots \otimes T^{x_n})\mathbf{f} \in \mathcal{A}\}$$

where

$$T^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad T^1 = \begin{pmatrix} 1 & 0 \\ 0 & \exp(i\pi/4) \end{pmatrix}$$

[Cai, Lu, Xia 2017]

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Quantum states and entanglement

A state of n qubits (quantum bits) is described by a vector in

$$\underbrace{\mathbb{C}^2 \otimes \ldots \otimes \mathbb{C}^2}_{n} \cong \mathbb{C}^{2^n}$$

A quantum state of multiple qubits is fully entangled if it cannot be written as a tensor product.

For example:

- $(1, 0, 0, 1)$ is fully entangled
- $(0, 0, 0, -1, 0, 1, 0, 0) = (0, -1, 1, 0) \otimes (0, 1)$ is not fully entangled

Entanglement classification under SLOCC

SLOCC = ‘stochastic local operations and classical communication’

Define an equivalence relation on vectors in $\mathbb{C}^{2^n}$:

$$\mathbf{f} \sim \mathbf{g} \quad \Longleftrightarrow \quad \mathbf{f} = (A_1 \otimes \ldots \otimes A_n)\mathbf{g}$$

for some invertible 2 by 2 matrices $A_1, \ldots, A_n$

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Fully entangled three-qubit states: If $\mathbf{f} \in \mathbb{C}^{2^3}$ is fully entangled, then either

- $\mathbf{f} \sim \mathrm{EQ}_3$, or
- $\mathbf{f} \sim \mathrm{ONE}_3$

Correspondences

Counting complexity theory	**Quantum information theory**
function	quantum state
degenerate function	product state
non-degenerate function	entangled state
affine function	stabiliser state
EQ_3	GHZ state
ONE_3	W state
holographic transformation	stochastic local operation with classical communication (SLOCC)
Holant problem	(strong) classical simulation of quantum circuits

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The dichotomy for HOLANT^{+}

Theorem

Let $\mathcal{F}$ be a set functions, then HOLANT$^{+}(\mathcal{F})$ = HOLANT$(\mathcal{F} \cup \{\delta_0, \delta_1, \delta_+, \delta_-\})$ is #P-hard unless $\mathcal{F}$ is in one of the following tractable families:

- unary and binary functions

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- affine functions

$$\mathcal{F} \subseteq \mathcal{A}.$$

Proof approach

There is a full dichotomy for HOLANT $(\{f\} \mid \{g\})$, where f is a symmetric ternary function and g is a symmetric binary function

[Cai, Huang, Lu 2012]

To get a dichotomy for HOLANT$^+$ $(\mathcal{F})$:

- Assume $\mathcal{F}$ is not one of the tractable families
- Then there exists a fully entangled function in $\langle \mathcal{F}, \delta_0, \delta_1, \delta_+, \delta_- \rangle$ which has arity ≥ 3
- Find $f, g \in \langle \mathcal{F}, \delta_0, \delta_1, \delta_+, \delta_- \rangle$ with arity(f) $= 3$ and arity(g) $= 2$, which are symmetric and fully entangled
- Reduce from HOLANT $(\{f\} \mid \{g\})$ to show hardness

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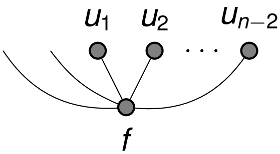
Gadget constructions from quantum theory

Theorem (Popescu & Rohrlich, 1992; Gachechiladze & Gühne, 2017)

Let f be a fully entangled n-ary function. Then, for any two of the inputs, there exists some fully entangled binary gadget over $\{f, \delta_0, \delta_1, \delta_+, \delta_-\}$.



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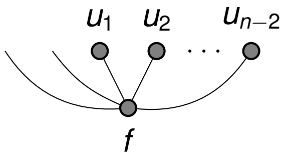
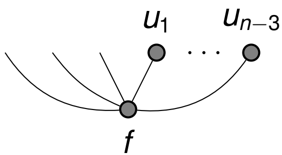
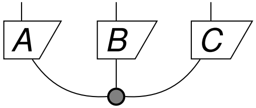


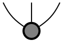
Theorem
Let f be a fully entangled n-ary function with $n \geq 3$. Then there exists a fully entangled ternary gadget over $\{f, \delta_0, \delta_1, \delta_+, \delta_-\}$ for some choice of three of the inputs.



‘Virtual gadgets’

It can be convenient to think of functions as gadgets even if they are not defined that way, e.g. for a fully entangled ternary function:



where  is assigned $\begin{cases} \mathrm{EQ}_3 & \text{or} \\ \mathrm{ONE}_3 \end{cases}$

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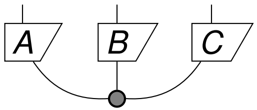
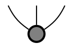


where  is assigned $\begin{cases} \mathrm{EQ}_3 & \text{or} \\ \mathrm{ONE}_3 \end{cases}$

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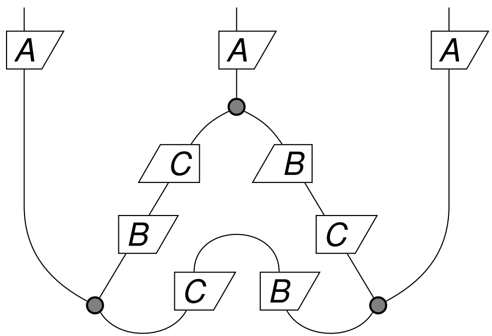

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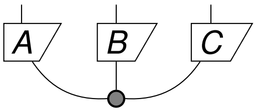
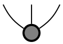


where  is assigned $\begin{cases} EQ_3 & \text{or} \\ ONE_3 \end{cases}$

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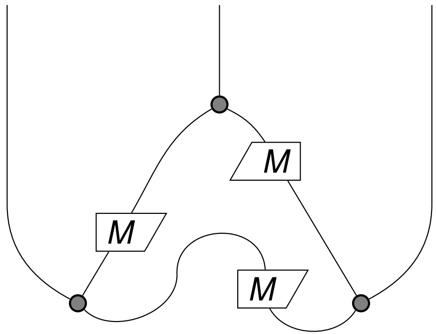
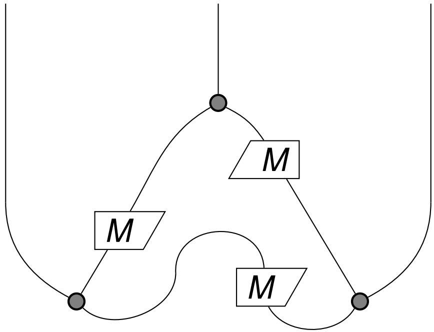


Gadgets for symmetric entangled ternary functions



Given a set $\mathcal{F}$ containing a fully entangled ternary function, can show with a bit of effort:

- either it is possible to realise a symmetric fully entangled ternary function over $\mathcal{F} \cup \{\delta_0, \delta_1, \delta_+, \delta_-\}$ using the above gadget (possibly with some extra effort), or
- $\mathcal{F} \cup \{\delta_0, \delta_1, \delta_+, \delta_-\}$ is one of the known tractable families

The dichotomy for $\textsc{Holant}^c$

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Let $\mathcal{F}$ be a set of functions, then $\textsc{Holant}^c(\mathcal{F}) := \textsc{Holant}(\mathcal{F} \cup \{\delta_0, \delta_1\})$ is #P-hard unless $\mathcal{F}$ is in one of the following tractable families:

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- 'transformable to affine' functions: $\exists M$ such that $M^T \circ \{\text{EQ}_2, \delta_0, \delta_1\} \subseteq \mathcal{A}$ and $\mathcal{F} \subseteq M \circ \mathcal{A}$
- local affine functions.

Sketch of hardness proof

Combine methods from real-valued Holant^c dichotomy [Cai, Lu, Xia 2017] and Holant$^+$ dichotomy [B 2017]

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- Use pinning and self-loops to reduce arity while preserving properties of being fully entangled and having arity ≥ 3
- Either get ternary function, or get 4-ary function which is non-zero only on inputs 0000, 0011, 1100, 1111 (up to permutation of arguments)

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- Pick a function $f \in \langle \mathcal{F}, \delta_0, \delta_1 \rangle$ with arity$(f) \geq 3$ which is fully entangled
- Use pinning and self-loops to reduce arity while preserving properties of being fully entangled and having arity ≥ 3
- Either get ternary function, or get 4-ary function which is non-zero only on inputs 0000, 0011, 1100, 1111 (up to permutation of arguments)
  - With ternary function, can show hardness using techniques from HOLANT$^+$ dichotomy

Sketch of hardness proof

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  - With 4-ary function, can reduce from $\#\text{CSP}_2^c$

Outline

Introduction to holant problems

Basic quantum information theory

The new results

Conclusions

Summary and outlook

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Thank you.