MAS228 Probability II

Test 8 November 2007

The duration of this test is 45 minutes. Write your name and student number in the spaces below.

Electronic calculators may be used. The calculator must not be preprogrammed prior to the examination. Enter the name and type here _____

Answer all questions. Write all answers in the spaces provided. If you run out of space for an answer, continue on the back of the page.

Name: _____

Student Number: _____

1. X and Y are independent random variables with $X \sim Bernoulli\left(\frac{1}{2}\right)$ and $Y \sim Bernoulli\left(\frac{1}{3}\right)$.

   (a) [4marks] Write down the probability generating functions $G_X(t)$ and $G_Y(t)$.

   (b) [4marks] Obtain the probability generating function of $Z = X + Y$.

   (c) [10marks] Obtain the probability mass function for Z.

2. Let X be a random variable with probability generating function

$$G_X(t) = (2-t)^{-1}e^{(t-1)}$$

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(a) 10marks Differentiate the p.g.f. to obtain $E[X]$ and $Var(X)$.

(b) 6marks Find $P(X=0)$, $P(X=1)$ and $P(X=2)$.

3.

(a) 10marks Kermit has £50 to gamble at a casino on a roulette wheel which has one zero. Each time he bets a stake of £10 on 'odds'. The probability of winning is $\frac{18}{37}$ for each game. If he wins a game he gets his £10 stake back plus an additional £10. If he loses he receives nothing (his stake is lost). He stops playing when he either goes broke (has £0) or reaches £100.

Calculate the probability that he reaches his target of £100.

(b) 10marks Gonzo enters a lift on floor 4 and wants to go to the penthouse. There are eleven floors labelled $0, 1, ..., 10$, where floor 0 is the ground floor and floor 10 is the penthouse. Each time he presses any lift button the lift either moves up 1 floor with probability $\frac{1}{2}$ or down 1 floor with probability $\frac{1}{2}$ regardless of the button he pressed. He continues pressing buttons until he either reaches the penthouse and alights or reaches the ground floor (when he gives up and leaves).

Calculate the probability that he reaches the penthouse.

4. 12marks Amanda plays a series of independent games. At each game she throws two dice. If she gets two sixes (double 6) she wins £10 and stops playing (so the series of games ends). If she gets any other outcome she continues playing. If the outcome is a double, but not a double 6, she wins £1. If she does not get a double she has to pay £C. Find the expected amount she has won by the end of the series of games. Find the value of £C so that the expected amount she has won is zero.

5.

(a) [10marks] The number of items X bought by a customer on a specific day has mean $\mu = 3$ and variance $\sigma^2 = 1$. This distribution is the same for all customers, and customers act independently. The number of customers N in the store on a particular day has $E[N] = 500$ and $Var(N) = 100$. If Y is the total number of items bought on that day, find $E[Y]$ and $Var(Y)$.

(b) [10marks] A branching process begins with a single ancestor forming generation zero. In each generation the number of offspring X of an individual has p.m.f. $P(X = x) = \frac{1}{4}$ for $x = 0,1,2,3$. Find the probability that the population will eventually die out.

6.

(a) [7marks] A random variable X has p.d.f. $f_X(x) = \theta e^{-\theta x}$ for $x > 0$. The p.d.f. is zero elsewhere. Find the moment generating function $M_X(t)$ and specify the range of t for which $M_X(t)$ is finite.

(b) [7marks] A random variable X has moment generating function $M_X(t) = \left(1 - t^2\right)^{-1}$. Obtain $E[X]$ and $Var(X)$.