MAS228 Probability II Test 9^{th} November 2006

1. X and Y are independent random variables with $X \sim Bernoulli(1/2)$ and $Y \sim Bernoulli(1/4)$.
   (a) (4 points) Write down the probability generating functions $G_X(t)$ and $G_Y(t)$.
   (b) (4 points) Obtain the probability generating function of $Z = X + Y$.
   (c) (10 points) Obtain the probability mass function for Z.

2. Let X be a random variable with probability generating function $G_X(t) = \frac{1}{2}(1+t)e^{(t-1)}$.
   (a) (10 points) Differentiate the p.g.f. to obtain $E[X]$ and $Var(X)$.
   (b) (6 points) Find $P(X = 0)$, $P(X = 1)$ and $P(X = 2)$.

3. Gonzo has £100. He plays roulette and at each game has the same stake. The probability of winning is p (and of losing is $q = 1 - p$) for each game. If he wins he receives double his stake and if he loses he receives nothing (his stake is lost). He stops playing when he either goes broke (has £0) or reaches £1000.
   (a) (10 points) Give the probability that he goes broke (in terms of q/p) if $p \neq \frac{1}{2}$ and his stake at each game is £1.
   (b) (10 points) Give the probability that he goes broke if $p = \frac{1}{2}$ and his stake at each game is £10.

4. (12 points) A mouse is placed at the centre of a maze with 3 paths. Two paths lead to a dead end so that he retraces the path back to the centre. The times taken (to follow the path and return to the centre) are 10 minutes and 12 minutes respectively. The third path leads out of the maze after 15 minutes. Each time the mouse is at the centre of the maze he chooses one of the three paths at random.

   Find $E[T]$ where T is the time in minutes until he gets out of the maze.

5. A population of amoebae begins with a single individual. In each generation an individual dies with probability $1/4$ (i.e. has no offspring) or has two offspring (by splitting in two) with probability $3/4$.
   (a) (10 points) Find the expected number of offspring in generation n.
   (b) (10 points) Find the probability that the population will die out eventually.

6. $X \sim Exp(2)$ so that the p.d.f. is $f_X(x) = 2e^{-2x}$ for $x > 0$. The p.d.f. is zero elsewhere.
   (a) (7 points) Show that the moment generating function is $M_X(t) = \left(1 - \frac{t}{2}\right)^{-1}$. State the constraint required on t.
   (b) (7 points) Obtain $E[X^r]$ for all positive integer values of r.