MTH5118 Probability II. Problem Sheet 8.

Please staple your coursework and post it in the Orange Box on the ground floor of the Maths building by **15:00** *on Wednesday 26th November 2008.*

1. X and Y have joint p.d.f. $f_{X,Y}(x,y) = 2$ for $0 < x < y < 1$ and $f_{X,Y}(x,y) = 0$ elsewhere. Find the joint p.d.f. for $U = Y - X$ and $V = X$, and hence find the marginal p.d.f. for U.

2. X and Y are independent with $X \sim Gamma(\theta, \alpha)$ and $Y \sim Gamma(\theta, \beta)$. Find the joint p.d.f. of $U = \frac{X}{Y}$ and $V = X + Y$. State whether U and V are independent and find their marginal p.d.f.'s.

3. Sally has a bus journey then a walk in order to get to work each morning. Her bus journey takes time $a + X$, where X measures the excess over the minimum journey time a. Her walk takes time $b + Y$ where Y measures the excess over the minimum walk time b. X and Y are independent $U(0,1)$. Then $Z = X + Y$ measures the excess total journey time above the minimum time of $a + b$. Let $U = X$. Find the joint p.d.f. of U and Z (which gives the joint p.d.f. of X and Z since $U = X$). Hence find the conditional p.d.f. for $X|Z = z$.

4. (a) If $f_U(u) = \theta e^{-\theta(u-\alpha)}$ for $\alpha < u < \infty$, show that $V = U - \alpha \sim Exp(\theta)$. Hence state $E[U]$ and $Var(U)$.

(b) A device contains two components working in parallel, so that the device continues working whilst at least one of the devices is still working. If X is the time until one of the components fail and Y is the time until both fail (so that the device stops working) then X and Y have joint p.d.f.

$$f_{X,Y}(x,y) = 2\theta^2 e^{-\theta(x+y)} \ for \ 0 < x < y < \infty.$$

Find the marginal p.d.f. for X and hence state the distribution, mean and variance of X.

Find the conditional distribution of $Y|X = x$. Obtain $E[Y|X]$ and $Var(Y|X)$. Hence find $E[Y]$ and $Var(Y)$. Use the result that $E[XY] = E[XE[Y|X]]$ to obtain $Cov(X,Y)$ and $\rho(X,Y)$.