

MTH5118 Probability II. Problem Sheet 7.

Please staple your coursework and post it in the Orange Box on the ground floor of the Maths building by **15:00** on Wednesday 19th November 2008.

1. In each of the following cases identify if X and Y are independent random variables and justify your result. For the cases where they are independent write down the marginal p.d.f. and range for each of X and Y and state the value of C .

(a) $f_{X,Y}(x, y) = C(x + y)$ for $0 < x < 1$ and $0 < y < 1$ and the p.d.f. is zero elsewhere.

(b) $f_{X,Y}(x, y) = C(1 + x + y + xy)$ for $0 < x < 1$ and $0 < y < 1$ and the p.d.f. is zero elsewhere.

(c) $f_{X,Y}(x, y) = Cxy$ for $x > 0$, $y > 0$ and $x + y < 1$ and the p.d.f. is zero elsewhere.

(d) $f_{X,Y}(x, y) = C\frac{xe^{-x}}{y^5}$ for $0 < x < \infty$ and $1 < y < \infty$ and the p.d.f. is zero elsewhere.

2. Let X and Y be independent each with chi-squared distributions with $X \sim \chi_n^2$ and $Y \sim \chi_m^2$. Write down the m.g.f.'s $M_X(t)$ and $M_Y(t)$. Hence obtain the m.g.f. of $Z = X + Y$. State the distribution of Z .

3. Let X and Y be independent exponential random variables each with parameter $\theta = 10$. State their m.g.f.'s, $M_X(t)$ and $M_Y(t)$

(a) Let $U = X + Y$. Obtain the m.g.f. of U by writing it in terms of the m.g.f.'s for X and Y . Hence state the distribution of U .

(b) Let $V = X - Y$. Obtain the m.g.f. of V by writing it in terms of the m.g.f.'s for X and Y . Compare $M_V(t)$ with the m.g.f. for a double exponential (see Q3(a) of Problem Sheet 5). Hence state the p.d.f. of V .

4. Let X and Y be independent each with $N(0, 1)$ distribution. Let $U = a(X + Y)$ and $V = b(X + cY)$, where c is a constant and a and b are positive constants.

Find the mean and variance of each of U and V and the covariance $Cov(U, V)$.

Find a , b and c so that U and V each have zero mean and variance 1 and are uncorrelated.

Use the m.g.f. to obtain the marginal distributions of each of U and V for the values you have obtained for a , b and c .