MTH5118 Probability II. Problem Sheet 6.

Please staple your coursework and post it in the Orange Box on the ground floor of the Maths building by **15:00** *on Wednesday 12th November 2008.*

1. (a) Let $X \sim Exp(\theta)$. Use the standard transformation of variables result to obtain the p.d.f. for $Y = 1 - e^{-\theta X}$. State the distribution of Y.

(b) $X \sim N(0,1)$. Let $Y = |X|$, so that Y takes values on $[0, \infty)$. For $y > 0$, write the event $Y \leq y$ as an equivalent event for X. Hence find the c.d.f for Y, $F_Y(y) = P(Y \leq y)$, in terms of the c.d.f. for X. Differentiate $F_Y(y)$ to obtain the density function for Y.

2. Let $f_{X,Y}(x,y) = C$ for x, y such that $x > 0$, $y > 0$ and $x + y < 1$; $f_{X,Y}(x,y) = 0$ elsewhere. Find the value of C. Find the marginal p.d.f. for (i) X; (ii) Y. Let $Z = X + Y$. Find the c.d.f. $P(Z \leq z)$ for (a) $z \leq 0$; (b) $0 < z < 1$; (c) $z \geq 0$. Hence obtain the p.d.f. $f_Z(z)$.

3. A woman arrives at a bus stop at a random time between 12:00 and 13:00. A bus arrives at the bus stop at a random time between 12:00 and 14:00 (which is independent of the arrival time of the woman). If X and Y give the time in hours after 12.00 that the woman arrives and the bus arrives, then X and Y have joint p.d.f.

$f_{X,Y}(x,y) = \frac{1}{2}$ for $0 < x < 1,\ 0 < y < 2$ and $f_{X,Y}(x,y) = 0$ elsewhere.

(a) Write the event that the woman misses the bus as an event in terms of X and Y and hence find the probability of this event.

(b) Write the event that the woman catches the bus but has to wait at least 1 hour for it to arrive as an event in terms of X and Y and hence find the probability of this event.

4. Random variables X and Y have joint p.d.f. $f_{X,Y}(x,y) = C(x + xy)$ for $0 < x < 1$, $0 < y < 2$ and $f_{X,Y}(x,y) = 0$ elsewhere. Find C and obtain the marginal p.d.f.'s $f_X(x)$ and $f_Y(y)$. Find $P(X + Y \leq 1)$.

5. Random variables X and Y have joint p.d.f. $f_{X,Y}(x,y) = Ce^{-y}$ for $0 < x < y < \infty$ and $f_{X,Y}(x,y) = 0$ elsewhere.

(a) Find C and obtain the marginal p.d.f.'s $f_X(x)$ and $f_Y(y)$.

(b) Let $Z = Y - X$. Find $P(Z \leq z)$ for $z > 0$ (note that $P(Z \leq z) = 0$ if $z \leq 0$). Hence find the p.d.f. of Z.