MTH5118 Probability II. Problem Sheet 5.

Please staple your coursework and post it in the Orange Box on the ground floor of the Maths building by **15:00** *on Wednesday 29th October 2008.*

1. Let X be a continuous random variable with p.d.f. $f_X(x) = e^{-(x-\alpha)}$ for $x > \alpha$ and $f_X(x) = 0$ elsewhere.

(a) Show that, for $t < 1$, the m.g.f. for X is $M_X(t) = \frac{e^{\alpha t}}{(1-t)}$.

(b) Differentiate the m.g.f. to find $E[X]$ and $Var(X)$.

(c) Let $Y = (X - \alpha)$. Show that the m.g.f. $M_Y(t) = e^{-\alpha t} M_X(t)$ and hence obtain $M_Y(t)$ and state the distribution of Y.

Use results from lectures for $E[Y]$ and $Var(Y)$ and the relation that $X = \alpha + Y$ to confirm your results in part (a).

2. Let $X \sim N(\mu, \sigma^2)$. State the m.g.f. of X, $M_X(t)$. Write the m.g.f. of $Y = \frac{(X-\mu)}{\sigma}$, $M_Y(t)$, in terms of the m.g.f. for X. Hence find $M_Y(t)$ and state the distribution of Y.

Expand the m.g.f. for Y in a power series and hence show that $E[Y^{2r+1}] = 0$ for all $r = 0, 1, ...$ and find $E[Y^{2r}]$ for $r = 1, 2, ...$

3. X has p.d.f. $f_X(x) = \frac{\theta}{2} e^{-\theta|x|}$ for all $-\infty < x < \infty$ (i.e. $f_X(x) = \frac{\theta}{2} e^{-\theta x}$ for $x \geq 0$ and $f_X(x) = \frac{\theta}{2} e^{\theta x}$ for $x < 0$).

(a) Show that the m.g.f. is $M_X(t) = \left(1 - \frac{t^2}{\theta^2}\right)^{-1}$, for $-\theta < t < \theta$. You will need to split the integral into two ranges corresponding to $x \geq 0$ and $x < 0$.

(b) Use the m.g.f. to obtain $E[X]$ and $Var(X)$.

(c) Let $Y = |X|$, so that Y takes values on $[0, \infty)$. Find the m.g.f. for Y, for $t < \theta$, (you will again need to split the range) and hence state the distribution of Y.

4. Integrate by parts to show that $\Gamma(\alpha + 1) = \alpha\Gamma(\alpha)$ for $\alpha > 0$, where $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-x} dx$.