MTH5118 Probability II. Problem Sheet 4.

Please staple your coursework and post it in the Orange Box on the ground floor of the Maths building by **15:00** *on Wednesday 22nd October 2008.*

1. Consider the females in a society. The distribution of the number of offspring is the same for all females in all generations. Let N be the number of children born to a female and let X be the number of these children who are female. Each child has probability $\frac{1}{2}$ of being female independently of the other children. State the distribution of $X|N = n$.

If $P(N = 0) = \frac{1}{4}$ and $P(N = 2) = \frac{3}{4}$, use the theorem of total probability to find $P(X = x)$ for each value $x = 0, 1, 2$. Find $E[X]$ and $Var(X)$.

Let Y_n be the number of females who are n^{th} generation direct female descendants of a particular female. Find $E[Y_n]$ and $Var(Y_n)$.

Now consider 100 females. Find the mean and variance of the total number of female descendants in generation n from these females.

2. A population of amoebae begins with a single individual. In each generation, each individual dies with probability 1/3 or produces two 'offspring' (by splitting in two) with probability 2/3.

(a) Find the probability mass function of the number of amoebae in generation 2.

(b) Find the probability that the population will die out by generation 3.

(c) Find the probability that the population will eventually die out.

3. Consider a branching process, starting with one ancestor, where the number of offspring X for an individual has $P(X = x) = pq^x$ for $x = 0, 1, 2, \ldots$. Find $G_X(t)$. Determine the probability of eventual extinction for this process in terms of p.

4. The family surname in a certain society survives through the male line of descent. The number of male offspring X for a male has $P(X = 0) = 1/4$, $P(X = 1) = 1/4$ and $P(X = 2) = 1/2$. Find the probability θ that the male line of descent of a particular male will eventually die out.

At a certain time the surname Earwacker is given to K newborn males from the society in honour of a benefactor from abroad. No-one else in the society has that surname.

(i) If $K = 10$, find the probability that the surname Earwacker will eventually die out.

(ii) If K is a random variable use the theorem of total probability to show that the probability that the surname Earwacker will eventually die out is $G_K\left(\frac{1}{2}\right)$.